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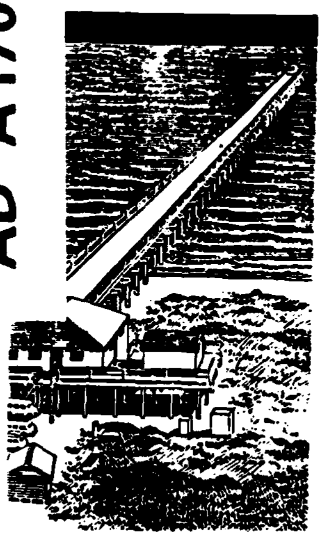

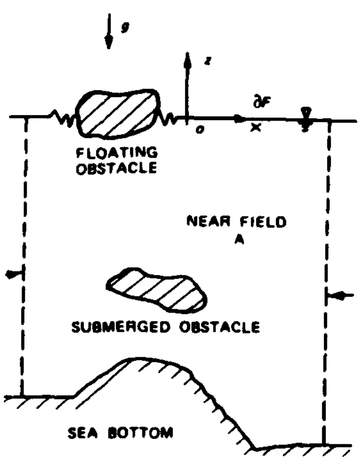

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INSTRUCTION REPORT CERC-86-3

WAVERTF: WAVE PROPAGATION OVER OBSTACLES AND IRREGULAR TOPOGRAPHY A USER'S MANUAL

by

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A hybrid element method is developed for the calculation of reflection, transmission, and hydrodynamic forces of water waves passing a finite near field involving obstacles and a sea bottom of irregular geometrical configuration. The obstacles may represent fixed floating and/or submerged marine structures, and the sea bottom may consist of undersea mounts, reefs, and trenches. In the far fields, water depths are assumed constant but not

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20. ABSTRACT (Continued).

necessarily equal. A variational principle with a proper functional is established such that the matching conditions are satisfied along the common boundaries of the near and far fields. Three examples are presented and compared with laboratory data and numerical and theoretical results. The WAVERTF computer program and user's manual are presented. An example problem is also included.

PREFACE

This report describes a hybrid element scheme for solving linear wave propagation over obstacles and irregular topography and provides a user's manual for the Wave Reflection, Transmission, and Forces (WAVERTF) Program. The research in this report was authorized by the Office, Chief of Engineers (OCE), Civil Works Research and Development, under Waves at Entrances Work Unit 31673, Harbor Entrances and Coastal Channels Program, at the Coastal Engineering Research Center (CERC) of the US Army Engineer Waterways Experiment Station (WES). Messrs. John H. Lockhart and John G. Housley, OCE, US Army Corps of Engineers, were the Technical Monitors.

The report was prepared by Dr. H. S. Chen and Ms. Willie A. Brown, Coastal Oceanography Branch (COB), CERC, under direct supervision of Dr. Edward F. Thompson, Chief, COB, and Mr. H. Lee Butler, Chief, Research Division, and under general supervision of Mr. Charles C. Calhoun, Jr., Assistant Chief, and Dr. John R. Houston, Chief, CERC. This report was edited by Ms. Shirley A. J. Hanshaw, Publications and Graphic Arts Division, WES.

Director of WES during the publication of this report was COL Allen F. Grum, USA. Technical Director was Dr. Robert. W. Whalin.

CONTENTS

CONVERSION FACTORS, NON-SI TO SI (METRIC) UNITS OF MEASUREMENT

Non-SI units of measurement used in this report can be converted to SI (metric) units as follows:

Multiply	By	To Obtain
feet	0.3048	metres
inches	2.54	centimetres

WAVERTF: WAVE PROPAGATION OVER OBSTACLES AND IRREGULAR TOPOGRAPHY
A USER'S MANUAL

PART I: MATHEMATICAL FORMULATION

1. In this study, a hybrid element method is developed for solving the progagation of linear water waves over a finite near field involving irregular obstacles and bathymetry. In the far fields water depths are assumed to be constant but not necessarily equal. A geometrical configuration of the problem is illustrated in Figure 1. The problem has been studied by numerous investigators over the years (Bai and Yeung 1974; Black, Mei, and Bray 1971; Chen and Mei 1974; Dean and Ursell 1959; Lamb 1945; Lee, Ayer, and Chiang 1980; Miles 1967; Mynett, Serman, and Mei 1979; and Newman 1965a,b). However, their problems are often oversimplified, and the solutions are not effective in application. Effective application of these problems can be found in Chen (1984) where the solutions are similar to those in Part I of this report.

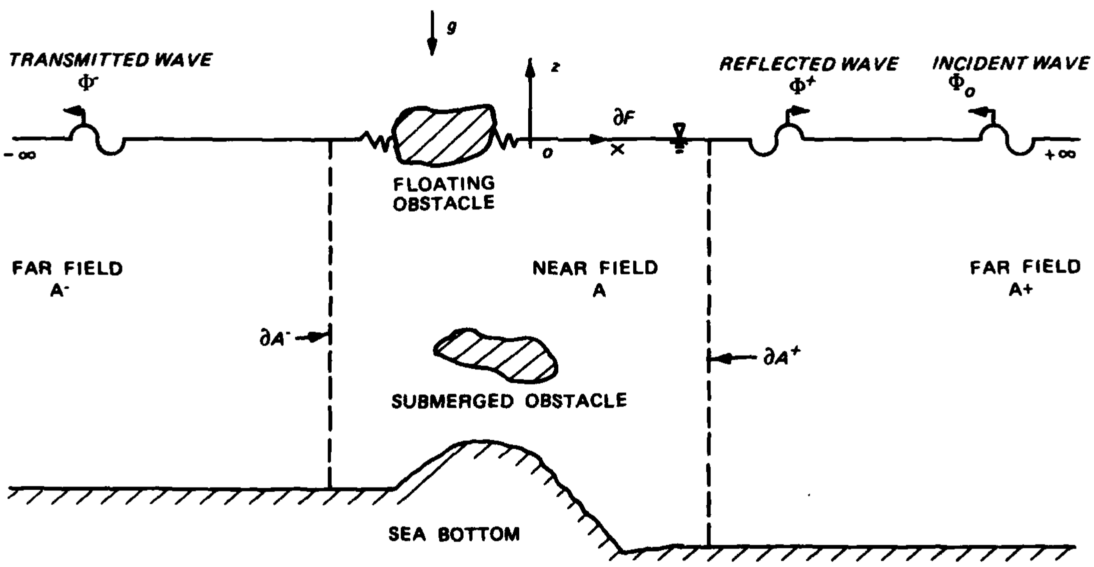

Figure 1. Geometrical configuration of the boundary value problem

Boundary Value Problem

2. Let (x,z)* be Cartesian coordinates with $z = 0$ representing the undisturbed water surface and with the upward direction being the positive z-axis, as shown in Figure 1. The fluid motion in a linear wave field is assumed to be two-dimensional such that the conservations of mass and momentum are written as

$$\frac{\partial u_j}{\partial x_j} = 0 \tag{1}$$

$$\frac{\partial u_j}{\partial t} = - \frac{\partial}{\partial x_j}\left(\frac{p}{\rho} + gz\right) - \varepsilon u_j \tag{2}$$

where

u_j = j-component flow velocity

$j=1,2$ = x and z components, respectively

t = temporal coordinate

p = pressure

ρ = water density

g = gravitational acceleration

3. In Equation 2 the friction term is analogous to Heaps (1969) and is modeled by $-\varepsilon u_j$ which is linearly proportional to the flow velocity. The coefficient ε is generally a spatial complex function, indicating spatial variability and phase difference from u_j. A more detailed explanation of ε will be given later.

4. For convenience, a potential function ϕ is defined such that

$$- \frac{\partial \phi}{\partial t} = \frac{p}{\rho} + gz \tag{3}$$

Since the wave is assumed to be sinusoidal in time with radian frequency ω, we may separate temporal and spatial dependencies by

---

* For convenience, symbols and abbreviations are listed in the Notation (Appendix C).

$$\begin{bmatrix} u_j(x,z,t) \\ \phi(x,z,t) \\ \zeta(x,z,t) \end{bmatrix} = \begin{bmatrix} U_j(x,z) \\ \Phi(x,z) \\ \eta(x,z) \end{bmatrix} \exp(-i\omega t) \tag{4}$$

where

ζ = the free surface displacement

U = spatial part of flow velocity

Φ = spatial part of the velocity potential function

$i = \sqrt{-1}$

η = free surface displacement

Substituting Equations 3 and 4 into Equation 2, we have

$$U_j = \lambda \frac{\partial \Phi}{\partial x_j} \tag{5}$$

where

$$\lambda = \frac{1}{1 + \frac{i\varepsilon}{\omega}} \tag{6}$$

Substituting Equations 4 and 5 into Equation 1, we have

$$\nabla \cdot \lambda \nabla \Phi = 0 \tag{7}$$

where ∇ is the two-dimensional gradient operator.

5. At the free surface $z = \zeta$, the atmospheric pressure is taken as the reference pressure $p = 0$; therefore, Equations 3 and 4 give the free surface displacement in terms of Φ .

$$\zeta = -\frac{1}{g}\frac{\partial \Phi}{\partial t} \quad \text{or} \quad \eta = \frac{i\omega}{g}\Phi \qquad \text{at} \quad z = 0 \tag{8}$$

The linearized free surface boundary condition is then obtained from the kinematic condition and Equation 8 to become

$$\lambda\frac{\partial \Phi}{\partial z} - \frac{\omega^2}{g}\Phi = 0 \qquad \text{at} \quad z = 0 \tag{9}$$

6. At the solid boundaries along the bottom and the stationary obstacles, an absorbing boundary condition is used.

$$\frac{\partial \Phi}{\partial n} - \alpha\Phi = 0 \qquad \text{at} \quad z = -h \quad \text{and} \quad \partial B \tag{10}$$

where n is the unit normal vector outward from the water region. α can be interpreted as the absorption coefficient. If the normal velocity vanishes, then $\alpha = 0$.

7. At $x \to \pm\infty$ in the far fields, the radiation conditions are required for the reflected and transmitted waves Φ^+ and Φ^- , respectively, to ensure a unique solution.

$$\frac{\partial \Phi^{\pm}}{\partial x} \pm ik_o^{\pm}\Phi^{\pm} = 0 \qquad \text{at } x \to \pm\infty \tag{11}$$

where k_o^+ and k_o^- are the wave numbers of the propagating modes in the far fields. The superscripts + and - from here on are referred to as the quantity in the region A^+ and A^- (see Figure 1).

8. Therefore, a boundary value problem is established with Equation 7 as the governing equation and Equations 9, 10, and 11 as the boundary conditions. Note that if there is no friction, $\varepsilon = 0$ and $\lambda = 1$. Equation 7 reduces to a Laplacian and Equations 9, 10, and 11 to their counterparts in the usual formulation of a linear wave problem.

Variational Principle and Hybrid Element Approximation

9. A hybrid element method is employed to solve the boundary value problem. The water domain is divided into three regions, A , A^+ , A^- , as shown in Figure 1. A conventional finite element approximation with a nodal-type element is used in the near field A , and analytical solutions with unknown coefficients are used to describe the far fields A^- and A^+ . A variational principle with a proper functional is established such that the matching conditions are satisfied at the common boundaries of the near and far fields. Calculations are thus localized in the near field for the nodal and coefficient unknowns.

10. The variational principle for the boundary value problem requires that the following functional Π be stationary with respect to the arbitrary first variation of Φ and $\Phi^{\pm}$.

$$\Pi(\Phi,\Phi^{\pm}) = \iint_A 1/2\lambda(\nabla\Phi)^2\, dA - \int_{\partial F} \frac{\omega^2}{2g}\,\Phi^2\, dS - \int_{\partial B} \lambda\alpha\Phi^2\, dS$$

$$- \int_{\partial A^-} \left(\frac{\Phi^-}{2} - \Phi\right)\lambda^- \frac{\partial\Phi^-}{\partial x}\, dS$$

$$+ \int_{\partial A^+} \left[\left(\frac{\Phi^+}{2} - \Phi\right)\lambda^+ \frac{\partial\Phi^+}{\partial x} - \Phi\lambda\frac{\partial\Phi_o}{\partial x} + \Phi_o\lambda^+ \frac{\partial\Phi^+}{\partial x} + \Phi_o\lambda^+ \frac{\partial\Phi_o}{\partial x}\right] dS \qquad (12)$$

In Equation 12, $\Phi_o(x,z)$ is the spatial part of the incident wave and is written as

$$\Phi_o(x,z) = -\frac{iga_o}{\omega}\,\frac{\cosh k_o^+(z + h^+)}{\cosh k_o^+ h^+}\,\exp\,(-ik_o^+ x) \qquad (13)$$

where

a_o = incident wave amplitude

k_o = wave number of the propagating mode

Naturally, the incident wave is coming from $x = +\infty$ and propagating to the left. Construction of the functional, Equation 12, and proof of the equivalency between the variational principle and the boundary value problem are achieved by procedures similar to those given by Chen and Mei (1974) and are not given in this report.

11. Triangular elements with linear shape function are employed to subdivide the near field. In the far fields, the friction terms are usually of minor concern in practice and are omitted. The bottoms are also assumed to be constant depths $h^{\pm}$ and impermeable such that $\alpha = 0$. Therefore, the analytical solutions of the outgoing waves in the far fields can be expressed as follows:

$$\Phi^{\pm} = -\frac{iga_o}{\omega} \sum_{m=0}^{\infty} R_m^{\pm}\,\frac{\cosh k_m^{\pm}(z + h^{\pm})}{\cosh k_m^{\pm} h^{\pm}}\,\exp\,(\pm ik_m^{\pm} x) \qquad (14)$$

where the coefficients $R_m^{\pm}$ are unknown constants to be determined and are the reflection and transmission coefficients when $m = 0$. $k_o^{\pm}$ and $k_m^{\pm}$ $(m \geq 1)$ are the wave numbers of the propagating and evanescent modes, respectively, and are determined from the dispersion relation $\omega^2 = gk \tanh kh$. In the

computation, the number of the term m in Equation 14 is usually truncated when the value of $\exp(\pm ik_m^{\pm}x)$ is the order of 10^{-4}.

12. Using the conventional finite element approximation in the near field and Equation 14 in the far fields, one can deduce a set of simultaneous linear equations for the nodal and coefficient unknowns upon extremization of the functional. The stiffness matrix is symmetric and stored in a packed matrix form of semibandwidth. Gaussian elimination is employed for a solution. Computations are very efficient.

Free Surface and Wave Forces

13. Once the solution is obtained for the velocity potential Φ in the near field, and the coefficient constants $R_m^{\pm}$ in the far fields, the free surface, ζ or η, is given by Equation 8. The hydrodynamic pressure p_d is given by Equation 3 or

$$p_d = -\rho \frac{\partial \Phi}{\partial t} = \rho i\omega\Phi \exp(-i\omega t) \tag{15}$$

In Equation 15 the hydrostatic pressure is excluded. The friction τ_j is given in Equation 2 by $-\varepsilon u_j$ or

$$\tau_j = -\rho\varepsilon u_j = -\rho\varepsilon\lambda \frac{\partial \Phi}{\partial x_j} \exp(-i\omega t) \tag{16}$$

The wave forces and moment on the obstacle are basically the integration of the hydrodynamic pressure and friction stresses along the boundary of the obstacle. Therefore, along the solid boundary, the total forces F_j and the moment M_j in the positive direction perpendicular to the (x,z) plane contributed from hydrodynamic pressure and friction are

$$F_j = \int_{\partial S} (p_d n_j - \tau_j)\, dS \tag{17}$$

$$\vec{M}_y = \int_{\partial S} \vec{r} \times (p_d\vec{n} - \vec{\tau})\, dS \tag{18}$$

where

$\vec{n} = (n_x, n_z)$ = unit normal outward from the solid boundary ∂S

$\vec{r}$ = distance vector from the reference center for calculating moment

$\vec{\tau} = (\tau_x , \tau_z)$

as given in Equation 16.

14. From Equations 8 and 15 through 18, the free surface η , the hydrodynamic pressure p_d , the friction τ_j , the total forces F_j , and the moment M_y are thus calculated.

Friction Terms

15. In light of the classical solution by Stokes for an oscillating flat plate in laminar flow (Schlichting 1968), in which the retarding force is proportional to the maximum velocity of the plate and has a phase difference of $\pi/4$ from the velocity of the plate, ε is expressed as

$$\varepsilon = \beta \sqrt{\nu\omega} \exp (i\gamma) \tag{19}$$

where the friction coefficient β and the phase difference γ are presumed to be dimensionless constants which may vary spatially. The inclusion of β and γ in Equation 19 as the empirical constants is intended for simplicity to account for the real situation in most engineering applications where the boundary and bottom are not smooth and flat. For the Stokes solution mentioned above, $\beta = 1$ and $\gamma = -\pi/4$. Substituting Equation 19 into Equation 6

$$\lambda = \frac{1}{1 + i\beta\sqrt{\frac{\nu}{\omega}}\, e^{i\gamma}} \tag{20}$$

Since Equation 19, strictly speaking, is not a universal expression for the friction coefficient, some other expression for ε might also work well. The key point for assessing the expression of ε will be simplicity, efficiency, and accuracy in application.

Results for Idealized Problems

16. Numerical results are presented for the propagation of water waves

over three different geometrical configurations. Very little is known about α in Equation 10, and, as usual, $\alpha = 0$ is taken in all of the following calculations.

17. Case I is a finite step, and its finite element network is shown in Figure 2. The calculated result for $\beta = 0$, as shown in Figure 3, agrees

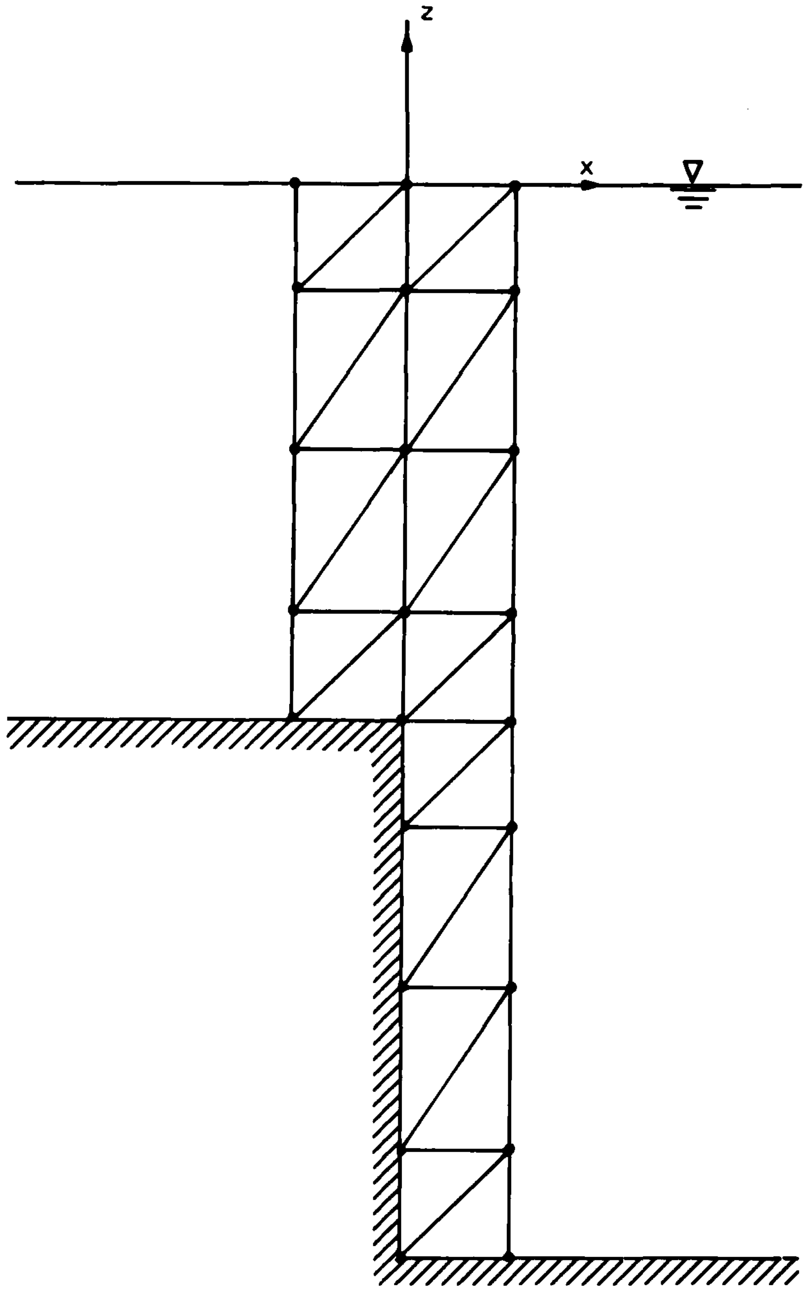

Figure 2. Finite element network of a finite step bottom

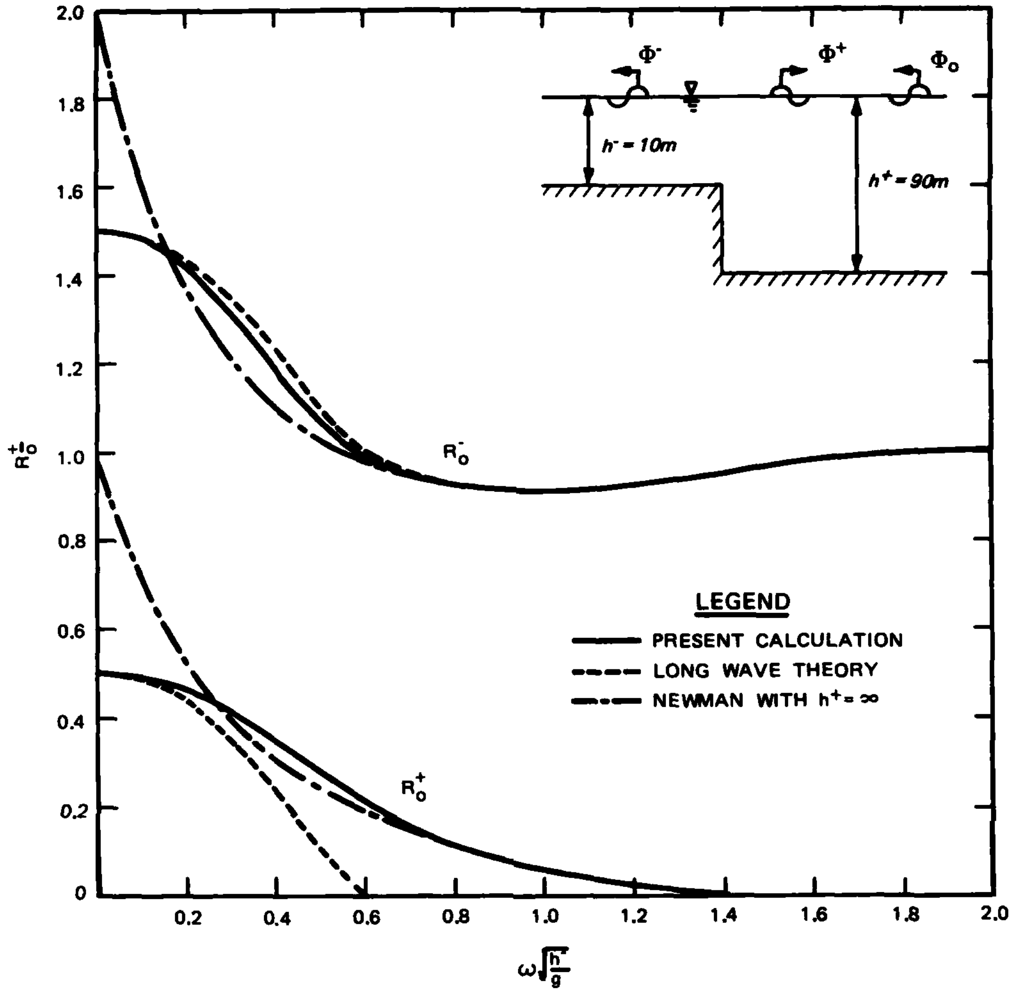

Figure 3. Reflection and transmission coefficients R_o^- and R_o^+ of the finite step bottom

well with the long wave theory (Lamb 1945) in the long wave range and the result of Newman (1965a) in the short wave range. Note that Newman's result is from the case of an infinite step where $h^+ = \infty$.

18. Case II is a fixed, semi-immersed circular cylinder in a finite water depth. Its finite element network is shown in Figure 4. The calculated results are shown in Figures 5* and 6 and agree fairly well with the experimental data and the theoretical results from Dean and Ursell (1959),

* A table of factors for converting non-SI units of measurement to SI (metric units is presented on page 3.

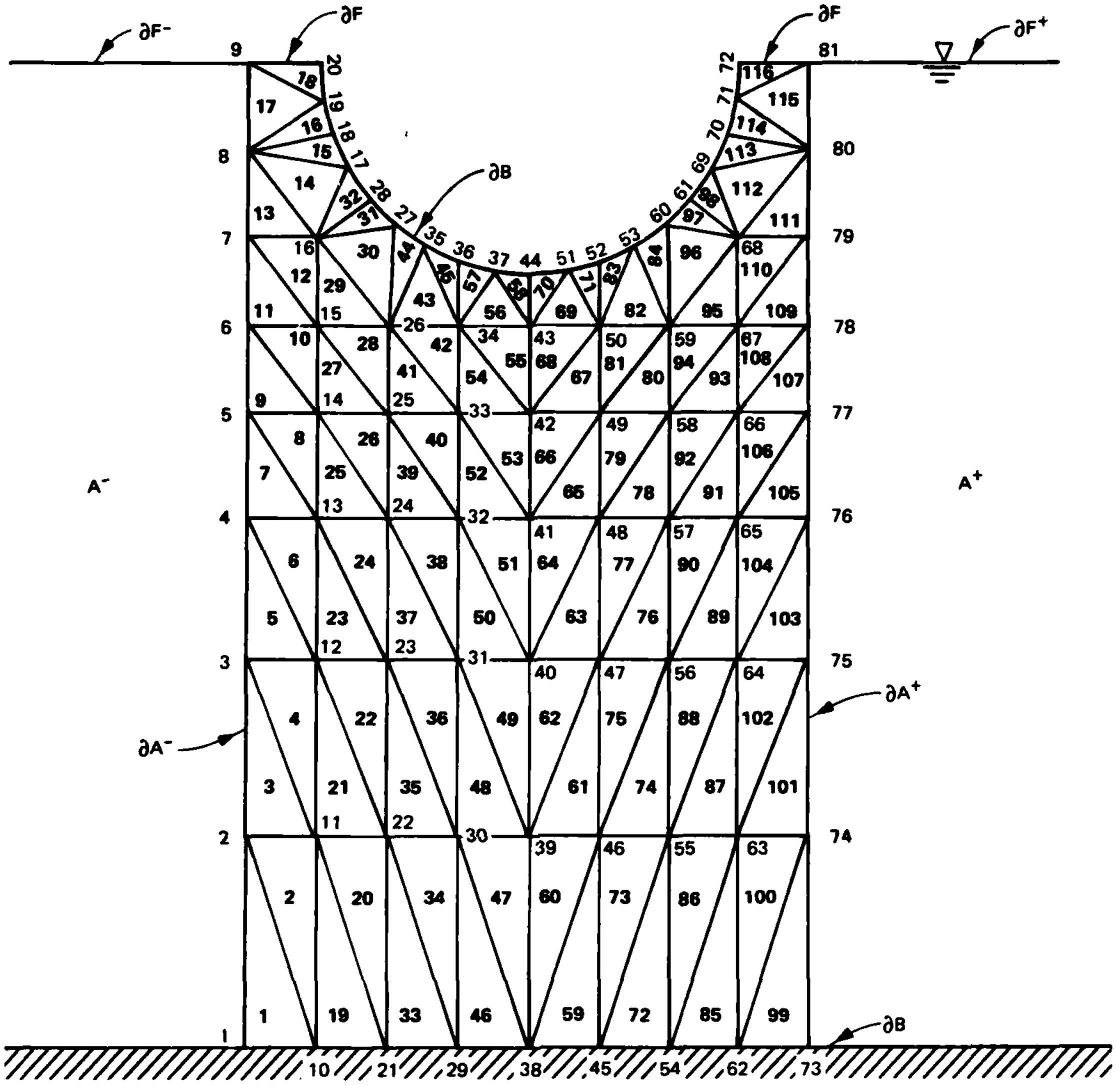

Figure 4. Finite element network of a fixed, semi-immersed circular cylinder in a finite water depth

particularly the results for $\beta = 10$. In Figure 6, the horizontal and vertical force coefficients f_x and f_z are defined as $\left(F_x\right)_{max}/(\rho g a_o a)$ and $\left(F_z\right)_{max}/(\rho g a_o a)$ where $\left(F_x\right)_{max}$ and $\left(F_z\right)_{max}$ are the maximum forces in the horizontal and vertical directions, respectively. In the computer program, the forces and moment coefficients are actually defined as

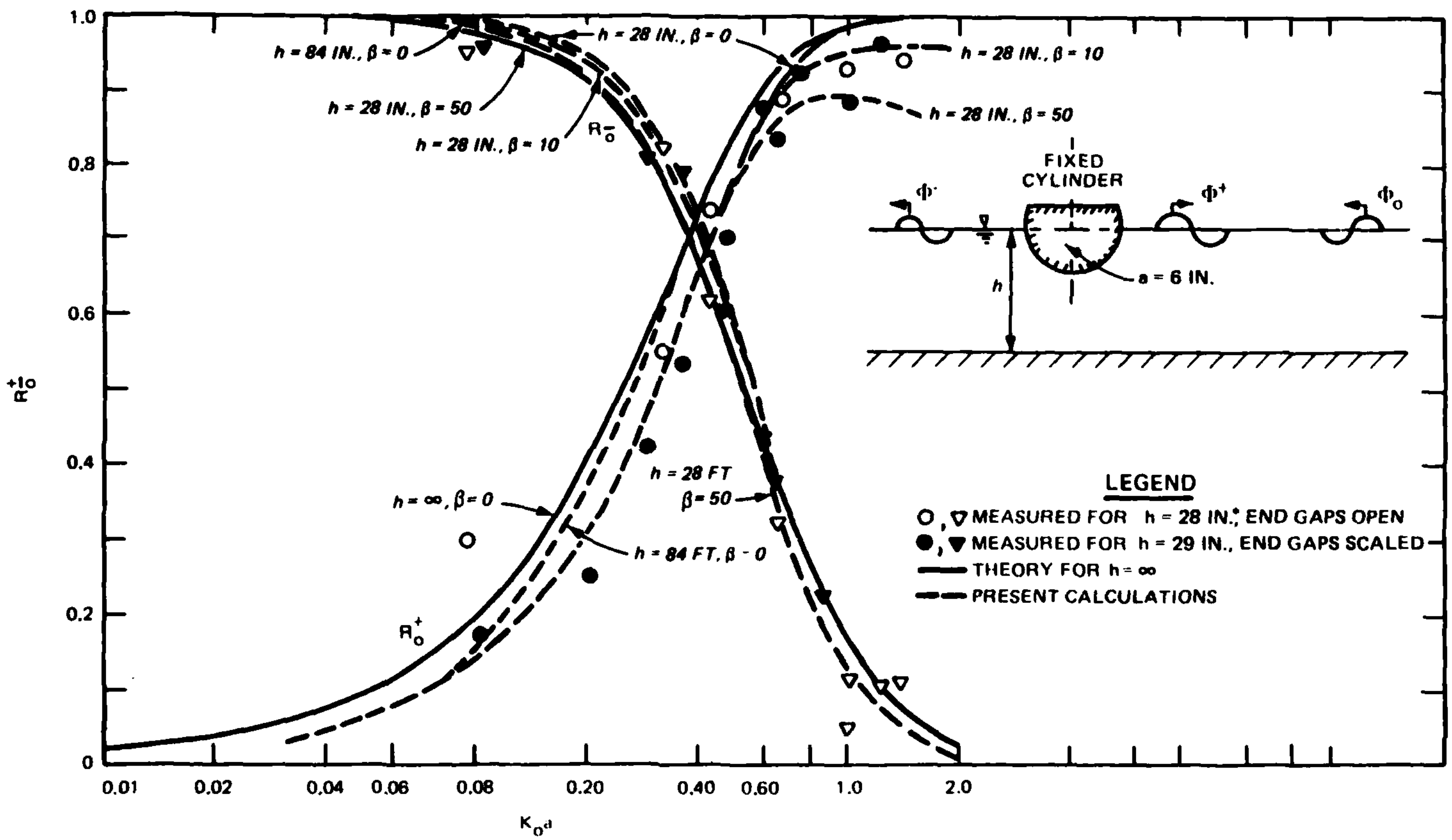

Figure 5. Comparison of reflection and transmission coefficients R+ and R_o^+ of the fixed, semi-immersed circular cylinder

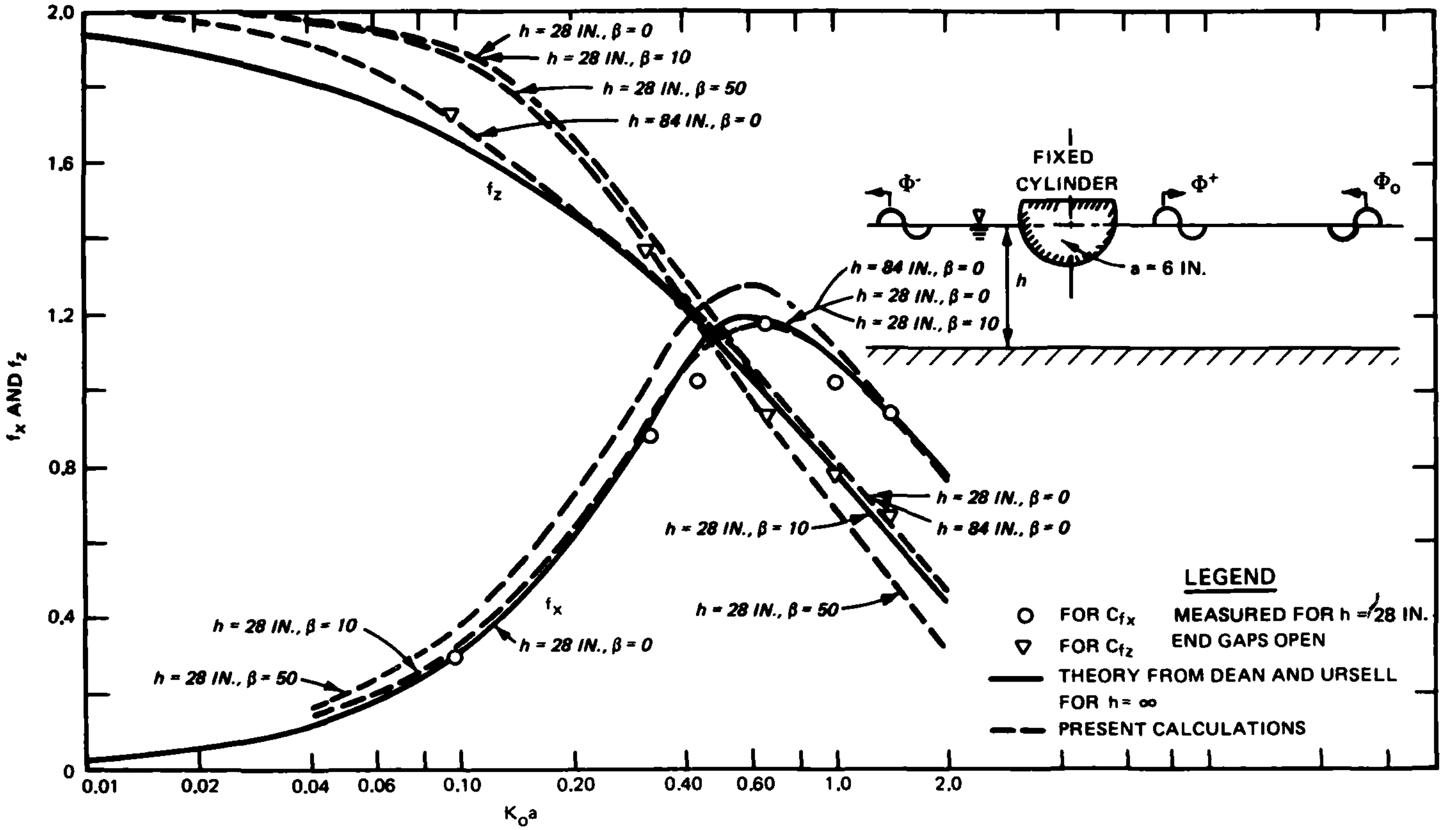

Figure 6. Comparison of horizontal and vertical force coefficients f_x and f_z of the fixed, semi-immersed circular cylinder

$$f_x = \frac{\left(F_x\right)_{max}}{\rho g a_o L} \tag{21}$$

$$f_z = \frac{\left(F_z\right)_{max}}{\rho g a_o L} \tag{22}$$

$$m_y = \frac{\left(M_y\right)_{max}}{\rho g a_o L^2} \tag{23}$$

where $\left(M_y\right)_{max}$ is the maximum moment of M_y in Equation 18, and L is the length scale for the force normalization which is the SCL(I) in the computer program given in Appendix B. $L = a$ is used in this calculation.

19. Note that the theoretical results of Dean and Ursell (1959) from the assumption of potential flow in an infinite water depth. In the calculations, the friction is applied only on the solid boundary and the bottom in the near field.

20. Case III is a rectangular trench. The calculated results for $\beta = 0$, as shown in Figure 7, agree quite well with the theoretical and calculated results and the experimental data by Lee, Ayer, and Chiang (1980).

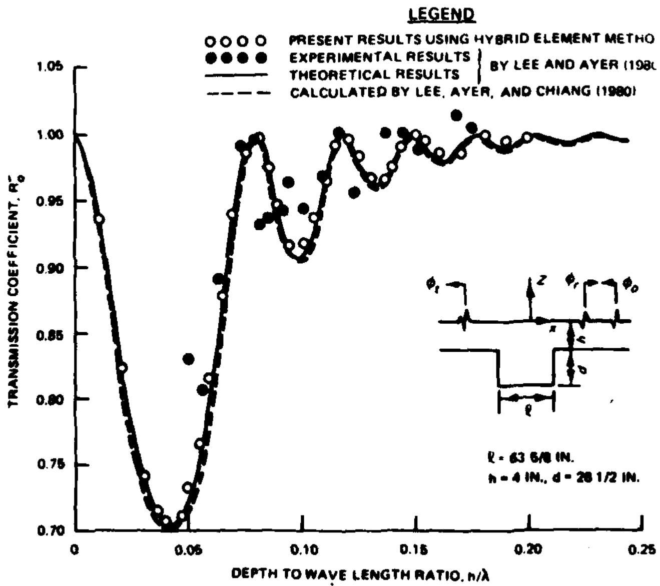

Figure 7. Comparison of transmission coefficient R_o^- of a rectangular trench

PART II: INPUT DATA PREPARATION AND STRUCTURE

General

21. A computer code WAVERTF program (listed in Appendix B), based on the hybrid element method as briefly described in Part I, is described in detail here. The computer code is implemented using FORTRAN V or FORTRAN Extended IV on the CYBER System. To use standard FORTRAN IV, changes to the program must be made. The PARAMETER statements must be taken out and the values introduced into the program separately. These values must also be placed in the DIMENSION statements. Metric (SI) units are used for all the physical quantities in the program. If other units are used, conversion factors must be implemented, mainly the viscosity (VISCO) and the gravitational constant (G) on card 00130. Note that a unit for mass is immaterial in the calculation.

Finite Element Grid

22. Some guidelines for correctly making finite element grids to run the computer program WAVERTF are given below. For clarity, we shall use the case of a fixed semi-immersed circular cylinder in a finite water depth, shown in Figure 4, as an example.

a. The incident wave is from $x = +\infty$. Therefore, the far field at the right hand side A^+ will be the reflected region, and at the left hand side A^- will be the transmitted region. Between the two far fields will be the near field A . For convenience, the origin of the Cartesian coordinates should be chosen somewhere at the free surface in region A.

b. Before starting the subdivision of the near field domain A into elements, the outer boundaries of the A domain, ∂A^+ and ∂A^- as shown in Figure 4, must be established to accommodate the obstacles and irregular bottom. The outer boundaries must be parallel to the vertical direction (z-axis).

c. Triangular elements are then used to subdivide the domain A . For best results the grid size, although arbitrary, should change gradually; and the horizontal extent of each grid size is generally not greater than one-tenth of the incident wavelength.

d. Numbering of the nodes requires several procedures.

   (1) For the nodes along ∂A^- , the numbering should start from the node at the bottom as 1 and sequentially increase along ∂A^- to the node at the free surface.

(2) For the nodes along ∂A^+ , the numbering should be such that the node at the free surface is equal to the total number of the nodes and sequentially decreases along ∂A^+ to the node at the bottom.

(3) For the nodes not at $\partial A^\pm$, the numbering, although arbitrary, should be such that the maximum value of the values of the nodal number difference in each element is as small as possible.

Input Data Structure

23. To run the computer program WAVERTF (which is listed along with the card numbers in Appendix B) the user must supply input data for a specific problem. The detailed requirements for these user inputs are given in this section. An example problem is described in Part III along with a listing of the corresponding input data.

24. There are only two PARAMETER statements, two DATA statements, and the subroutine DATAIN in the program that need to be modified for each new problem.

PARAMETER and DATA Statements

25. The numerical values in the following four statements in the program need be modified for each new problem:

```
00030  PARAMETER(NELE=116,NNOD=81,NBD=13,NKR=6,NKT=6,NBR=9,NBT=9)
00040  PARAMETER(NFMX=2,NSGF=2,NBODMX=19,NSGB=1)
00120  DATA NNODOT,NKROT,NKTOT,IFORCE/0,1,1,1/
00130  DATA GAMA,VISCO,G,TOLR/-0.78539816,1.E-6,9.80621,1.E-4/
```

26. The PARAMETER statements consist of the following parameters:

Parameter Name	Definition
NELE	Total number of elements
NNOD	Total number of nodes
NBD	Bandwidth, which is the maximum value of NBP, NBT, and the maximum value of nodal number difference in each element. (The value of NBD usually cannot be easily prepared, and an estimated value should be used for the first run. The correct value of NBD will be in the output of each run. For example, in the output file in Appendix A, the correct value for NBD is given just before wave period information.)

(Continued)

Parameter Name	Definition
NKR	Total number of wave modes for reflection $\left(k_m^+ \text{ in Equation 14}\right)$
NKT	Total number of wave modes for transmission $\left(k_m^- \text{ in Equation 14}\right)$
NBR	Number of nodes along the reflection boundary ∂A^+
NBT	Number of nodes along the transmission boundary ∂A^-
NFMX	Maximum nodal number allowed in a free surface segment ∂F
NSGF	Number of segments of free surface ∂F
NBODMX	Maximum nodal number allowed in a solid body
NSGB	Number of bodies for force calculation

27. The DATA statements consist of the following parameters:

Data Name	Definition
NNODOT	Index for the nodal value output 0 for no print out NNOD for printing out all the nodal potential values
NKROT	Number of the reflection coefficients R_m^+ in Equation 14 to be printed out
NKTOT	Number of the transmission coefficients R_m^- in Equation 14 to be printed out
IFORCE	Index for force calculation IFORCE=0 for no force calculation (Otherwise force will be calculated.)
GAMA	γ in Equation 19 (This is the phase difference. If it is not known it is usually taken to be $-\pi/4$.)
VISCO	ν in Equation 19 (This is the water viscosity. If it is not known it is usually taken to be 10^{-4} .)

Subroutine DATAIN

28. The listing for subroutine DATAIN is as follows:

```
01980       SUBROUTINE DATAIN(IGO,OMGSG,OMGA,G)
01990 C  - - - - - - - - - - - - - - - - - - - - - - - - - - - -
02000 C  OMGSG=(WAVE FREQUENCY)**2/GRAVITY CONSTANT.
02010 C  - - - - - - - - - - - - - - - - - - - - - - - - - - - -
02020       READ(5,4) IGO,WAVT
02030     4 FORMAT(I10,F10.1)
```

```
02040       IF(IGO.EQ.0) RETURN
02050       WRITE(6,8) WAVT
02060     8 FORMAT(//20("--")/10X,"WAVE  PERIOD,    WAVT=",F8.4,"
02070      1SECONDS,/20("--")//)
02080       OMGA=6.2831853/WAVT
02090       OMGSG=OMGA*OMGA/G
02100       RETURN
02110       END
```

29. This subroutine requires the wave period WAVT be read in with unit of seconds as the wave parameter. If another type of wave parameter is chosen to be read in, some modified statements to calculate OMGA and OMGSG are needed in the subroutine. Also, the format statement must be changed to get the right units. OMGA is the radian wave frequency ω in Equation 4 and is $2\pi/\mathrm{WAVT}$; $\mathrm{OMGSG} = \omega^2/g$.

Data Card Structure

30. Data are read by the main program and the subroutine DATAIN from device #5 (TAPE5). The following data sets must all appear and be placed in order.

a. Data set 1.

```
00180       READ(5,8) TITLE
00190     8 FORMAT(20A4)
```

TITLE has a dimension of 20 and is reserved for the title of the job. It uses a single card on which the user types a title or identifying label anywhere using any or all of columns 1-80.

b. Data set 2.

```
00290       READ(5,12) (I,X(I),Z(I),BETA(I),J=1,NNOD)
00300    12 FORMAT(3(I4,3F6.0))
```

In this data set, node numbers, their x and z coordinates, and friction coefficients are read in. I is the nodal number, X(I) and Z(I) are the x and z coordinates of nodal I, respectively, and $\mathrm{BETA(I)} = \beta$ is the friction coefficient as seen in Equation 19. Each card contains information for three nodes. Coordinate values must be read in for all nodes. The number of cards read is NNOD/3 , plus 1 if there is a remainder.

c. Data set 3.

```
00160     2 FORMAT(20I4)
00370       READ(5,2) (J,(ICON(I,J),I=1,3),L=1,NELE)
```

In this data set, element numbers and their nodal connectivities are read in. J is the element number, and ICON(I,J) are the nodal connectivities surrounding element J. Each card contains information for five elements. Nodal numbers must be read in counterclockwise order for all elements. The number of cards read in is NELE/5, plus 1 if there is a remainder.

d. Data set 4.

```
00160     2 FORMAT(20I4)
00610       DO 35 I=1,NSGF
00620       READ(5,2) JJ,(INF(J,I), J=1,JJ)
00680    35 CONTINUE
```

This data set is repeated NSFG times to account for NSFG segments on the free surface ∂F . In this data set, the number of nodes on each segment of the free surface, along with the nodal connectivity on the segment, are read in. JJ is the number of nodes on the segment of the free surface, and INF(J,I) is the nodal connectivity which must be in the order of from left to right.

e. Data set 5.

```
00160     2 FORMAT(20I4)
00170     4 FORMAT(8F10.4)
00820       DO 45 I=1,NSGB
00830       READ(5,2) JJ,(INBOD(J,I),J=1,JJ)
00840       READ(5,4) SCL(I),XC(I),ZC(I)
00910    45 CONTINUE
```

If NSGB=0, this data set is skipped. This data set is repeated NSGB times to account for NSGB segments on the solid boundaries ∂B on which the forces are to be calculated. On card 00820, the number of nodes and the nodal connectivities around each segment of the solid boundaries is read in. JJ is the number of nodes on the segment, and INBOD(J,I) is the nodal connectivity which must be in the sequence to make the water domain area positive (the water domain must be on the right hand side in the direction of nodal connectivity). Twenty values can be read from each card. Card 00830 should be immediately followed by card 00840. On this card the length scale SCL(I), which is used to normalize the forces, and XC(I) and ZC(I), which are the reference center for moment M_y , are read in. I is the segment of the solid boundary.

f. Data set 6.

```
01980         SUBROUTINE DATAIN(IGO,OMGSG,OMGA,G)
02020         READ(5,4) IGO,WAVT
02030     4   FORMAT(I10,F10.1)
```

As mentioned in paragraphs 28 and 29, this data set consists of

a set of cards containing an integer number for the sequential number of wave periods and a wave period in seconds indicated by IGO and WAVT, respectively. IGO should never be equal to 0 .

g. Data set 7.

At least one card with 0 typed in column 10 should be read in to indicate the end of all the input data.

31. All printed output is directed to device #6 (tape 6).

PART III: EXAMPLE PROBLEM

32. As an example a fixed, semi-immersed circular cylinder in a finite water depth (as shown in Figure 4), along with the finite element mesh, will be used here. For clarity, the length unit in Figures 4 through 6 is replaced by metres instead of inches in this calculation. In the water domain (A), there are 81 nodes and 116 elements. There exists one fixed floating obstacle. There are two free surfaces, one along nodes 9 and 20 and the other along nodes 72 and 81. To run this the program, the values in the two PARAMETER and two DATA statements in the main program (given in paragraph 33) need be changed along with the input data in paragraph 34. The output data are given in Appendix A.

PARAMETER and DATA Statements

33. In the main program the necessary changes needed to implement this particular problem are found in the PARAMETER statements and the DATA statements. The values are as follows:

```
00030  PARAMETER(NELE=116,NNOD=81,NBD=13,NKR=6,NKT=6,NBR=9,NBT=9)
00040  PARAMETER(NFMX=2,NSGF=2,NBODMX=19,NSGB=1)
00120  DATA NNODOT,NKROT,NKTOT,IFORCE/81,6,6,1/
00130  DATA GAMA,VISCO,G,TOLR/-0.78539816,1.E-6,9.80621,1.E-4/
```

Input Data File

34. The input file is given in the following list with accompanying card numbers. The title is on card 00010. All subsequent cards are assumed to have exactly 80 columns. Cards 00020 through 00280 represent the nodes and their x and z coordinates and friction coefficients. The elements and their nodal connectivities are on cards 00290 through 00520. Cards 00530 and 00540 have the number of nodes and the nodal connectivities on each segment of free surface. Card 00550 has the number of nodes and the nodal connectivities on each segment of the solid body for force calculation. Card 00560 has the length scale and the reference center chosen to be the radius and the center of the semicircular cylinder. Cards 00570 and 00580 have the index and the wave period. Card 00590 has a zero to represent the end of input.

Input File Listing

```
REFLECTION, TRANSIMISSION, AND WAVE FORCES FOR LINEAR WAVE
  1  -8.0 -28.0  0.00   2  -8.0 -22.0  0.00   3  -8.0 -17.0  0.00
  4  -8.0 -13.0  0.00   5  -8.0 -10.0  0.00   6  -8.0  -7.5  0.00
  7  -8.0  -5.0  0.00   8  -8.0  -2.5  0.00   9  -8.0   0.0  0.00
 10  -6.0 -28.0 10.00  11  -6.0 -22.0  0.00  12  -6.0 -17.0  0.00
 13  -6.0 -13.0  0.00  14  -6.0 -10.0  0.00  15  -6.0  -7.5  0.00
 16  -6.0  -5.0  0.00  17-5.196-3.000 10.00  18-5.638-2.052 10.00
 19-5.909-1.042 10.00  20  -6.0   0.0 10.00  21  -4.0 -28.0 10.00
 22  -4.0 -22.0  0.00  23  -4.0 -17.0  0.00  24  -4.0 -13.0  0.00
 25  -4.0 -10.0  0.00  26  -4.0  -7.5  0.00  27-3.857-4.596 10.00
 28-4.596-3.857 10.00  29  -2.0 -28.0 10.00  30  -2.0 -22.0  0.00
 31  -2.0 -17.0  0.00  32  -2.0 -13.0  0.00  33  -2.0 -10.0  0.00
 34  -2.0  -7.5  0.00  35-3.000-5.196 10.00  36-2.052-5.638 10.00
 37-1.042-5.909 10.00  38   0.0 -28.0 10.00  39   0.0 -22.0  0.00
 40   0.0 -17.0  0.00  41   0.0 -13.0  0.00  42   0.0 -10.0  0.00
 43   0.0  -7.5  0.00  44   0.0  -6.0 10.00  45   2.0 -28.0 10.00
 46   2.0 -22.0  0.00  47   2.0 -17.0  0.00  48   2.0 -13.0  0.00
 49   2.0 -10.0  0.00  50   2.0  -7.5  0.00  51 1.042-5.909 10.00
 52 2.052-5.638 10.00  53 3.000-5.196 10.00  54   4.0 -28.0 10.00
 55   4.0 -22.0  0.00  56   4.0 -17.0  0.00  57   4.0 -13.0  0.00
 58   4.0 -10.0  0.00  59   4.0  -7.5  0.00  60 3.857-4.596 10.00
 61 4.596-3.857 10.00  62   6.0 -28.0 10.00  63   6.0 -22.0  0.00
 64   6.0 -17.0  0.00  65   6.0 -13.0  0.00  66   6.0 -10.0  0.00
 67   6.0  -7.5  0.00  68   6.0  -5.0  0.00  69 5.196-3.000 10.00
 70 5.638-2.052 10.00  71 5.909-1.042 10.00  72   6.0   0.0 10.00
 73   8.0 -28.0  0.00  74   8.0 -22.0  0.00  75   8.0 -17.0  0.00
 76   8.0 -13.0  0.00  77   8.0 -10.0  0.00  78   8.0  -7.5  0.00
 79   8.0  -5.0  0.00  80   8.0  -2.5  0.00  81   8.0   0.0  0.00
  1   1  10   2   2   2  10  11   3   2  11   3   4   3  11  12   5   3  12   4
  6   4  12  13   7   4  13   5   8   5  13  14   9   5  14   6  10   6  14  15
 11   6  15   7  12   7  15  16  13   7  16   8  14   8  16  17  15   8  17  18
 16   8  18  19  17   8  19   9  18   9  19  20  19  10  21  11  20  11  21  22
 21  11  22  12  22  12  22  23  23  12  23  13  24  13  23  24  25  13  24  14
 26  14  24  25  27  14  25  15  28  15  25  26  29  15  26  16  30  16  26  27
 31  16  27  28  32  16  28  17  33  21  29  22  34  22  29  30  35  22  30  23
 36  23  30  31  37  23  31  24  38  24  31  32  39  24  32  25  40  25  32  33
 41  25  33  26  42  26  33  34  43  26  34  35  44  26  35  27  45  34  36  35
 46  29  38  30  47  30  38  39  48  30  39  31  49  31  39  40  50  31  40  32
 51  32  40  41  52  32  41  33  53  33  41  42  54  33  42  34  55  34  42  43
 56  34  43  37  57  34  37  36  58  43  44  37  59  38  45  46  60  38  46  39
 61  39  46  47  62  40  39  47  63  40  47  48  64  41  40  48  65  41  48  49
 66  42  41  49  67  42  49  50  68  43  42  50  69  43  50  51  70  43  51  44
 71  51  50  52  72  45  54  55  73  45  55  46  74  46  55  56  75  47  46  56
 76  47  56  57  77  48  47  57  78  48  57  58  79  49  48  58  80  49  58  59
 81  50  49  59  82  50  59  53  83  52  50  53  84  53  59  60  85  54  62  63
 86  55  54  63  87  55  63  64  88  56  55  64  89  56  64  65  90  57  56  65
 91  57  65  66  92  58  57  66  93  58  66  67  94  59  58  67  95  59  67  68
 96  60  59  68  97  60  68  61  98  61  68  69  99  62  73  74 100  63  62  74
101  63  74  75 102  64  63  75 103  64  75  76 104  65  64  76 105  65  76  77
106  66  65  77 107  66  77  78 108  67  66  78 109  67  78  79 110  68  67  79
111  68  79  80 112  68  80  69 113  69  80  70 114  70  80  71 115  71  80  81
116  72  71  81
  2   9  20
  2  72  81
 12  20  19  18  17  28  27  35  36  37  44  51  52  53  60  61  69  70  71  72
        6.0       0.0       0.0
          1        12.0
          2         6.0
          0
/
```

REFERENCES

Bai, K. J., and Yeung, R. 1974. "Numerical Solutions of Free Surface And Flow Problems," Proceedings of the 10th Symposium on Naval Hydrodynamics, Office of Naval Research, pp 609-641.

Black, J. L., Mei, C. C., and Bray, M. C. G. 1971. "Radiation and Scattering of Water Waves by Rigid Bodies," Journal of Fluid Mechanics, Vol 46, pp 151-164.

Chen, H. S. 1984. "Calculation of Water Wave Scattering With Friction," Proceedings of the 5th Engineering Mechanics Division Specialty Conference, American Society of Civil Engineers, pp 716-719.

Chen, H. S., and Mei, C. C. 1974 (Aug). "Oscillations and Wave Forces in an Offshore Harbor," Ralph M. Parson's Laboratory Report No. 190, Massachusetts Institute of Technology, Cambridge, Mass.

Dean, R. G., and Ursell, F. 1959 (Sep). "Interaction of a Fixed Semi-Immersed Circular Cylinder With a Train of Surface Waves," Hydrodynamics Laboratory TP No. 37, Massachusetts Institute of Technology, Cambridge, Mass.

Heaps, N. S. 1969 (Oct). "A Two-Dimensional Numerical Sea Model," Royal Society of London, Philosophical Transactions, Series A, Vol 265, No. 1160, pp 93-137.

Lamb, H. 1945. Hydrodynamics. 6th ed., Dover Publications, New York.

Lee, J. J., and Ayer, R. M. 1980. "Wave Propagation Over a Rectangular Trench," Journal of Fluid Mechanics, Vol 110, pp 335-347.

Lee, J. J., Ayer, R. M., and Chiang, W. L. 1980. "Interactions of Waves With Submarine Trenches," Proceedings of 17th International Coastal Engineering Conference, American Society of Civil Engineers, pp 812-822.

Miles, J. W. 1967. "Surface-Wave Scattering Matrix for a Shelf," Journal of Fluid Mechanics, Vol 28, pp 755-767.

Mynett, A. E., Serman, D. D., and Mei, C. C. 1979. "Characteristics of Salter's Cam for Extracting Energy From Ocean Waves," Applied Ocean Research, Vol 1, pp 13-20.

Newman, J. N. 1965a. "Propagation of Water Waves Over an Infinite Step," Journal of Fluid Mechanics, Vol 23, Part 2, pp 399-415.

________. 1965b. "Progagation of Water Waves Past Lone Two-Dimensional Obstacles," Journal of Fluid Mechanics, Vol 23, pp 23-29.

Schlichting, H. 1968. Boundary-Layer Theory, 6th ed., McGraw-Hill.

APPENDIX A: OUTPUT DATA FILE

The program with the correct PARAMETER and DATA statements and the input data file was submitted and run on a Control Data Cyber 170 Model 760 computer. Results in the output file are shown in the following pages of this appendix. In the output, nodal potential is given in the form of $\Phi/(iga_o/\omega)$, where iga_o/ω is the factor of the incident wave as given in Equation 13. The reflection and transmission coefficients are R_m^- and R_m^+ in Equation 14, and the wave forces and moment coefficients are f_x, f_z, and m_y as given in Equations 21, 22, and 23, respectively. The numerical results are self-explanatory. The CPU time for this run is less than 2 sec. The output file listing is as follows:

REFLECTION, TRANSIMISSION, AND WAVE FORCES FOR LINEAR WAVE

TOTAL NUMBER OF ELEMENTS, NELE= 116
TOTAL NUMBER OF NODES, NNOD= 81
TOTAL NUMBER OF EIGEN VALUES FOR REFLECTION, NKR= 6
TOTAL NUMBER OF EIGEN VALUES FOR TRANSMISSION, NKT= 6
MAX NODAL NO. ALLOWED IN A SEGMENT, NFMX= 2
INDEX FOR FORCE CALCULATION (NONE=0), IFORCE= 1
MAX NODAL NO. ALLOWED IN A BODY, NBODMX= 19

(X,Z) COORDINATE AND FRICTION COEF FOR EACH NOD

NOD	X	Z	FRIC	NOD	X	Z	FRIC	NOD	X	Z	FRIC
1	-8.0	-28.0	.0	2	-8.0	-22.0	.0	3	-8.0	-17.0	.0
4	-8.0	-13.0	.0	5	-8.0	-10.0	.0	6	-8.0	-7.5	.0
7	-8.0	-5.0	.0	8	-8.0	-2.5	.0	9	-8.0	.0	.0
10	-6.0	-28.0	10.0	11	-6.0	-22.0	.0	12	-6.0	-17.0	.0
13	-6.0	-13.0	.0	14	-6.0	-10.0	.0	15	-6.0	-7.5	.0
16	-6.0	-5.0	.0	17	-5.2	-3.0	10.0	18	-5.6	-2.1	10.0
19	-5.9	-1.0	10.0	20	-6.0	.0	10.0	21	-4.0	-28.0	10.0
22	-4.0	-22.0	.0	23	-4.0	-17.0	.0	24	-4.0	-13.0	.0
25	-4.0	-10.0	.0	26	-4.0	-7.5	.0	27	-3.9	-4.6	10.0
28	-4.6	-3.9	10.0	29	-2.0	-28.0	10.0	30	-2.0	-22.0	.0
31	-2.0	-17.0	.0	32	-2.0	-13.0	.0	33	-2.0	-10.0	.0
34	-2.0	-7.5	.0	35	-3.0	-5.2	10.0	36	-2.1	-5.6	10.0
37	-1.0	-5.9	10.0	38	.0	-28.0	10.0	39	.0	-22.0	.0
40	.0	-17.0	.0	41	.0	-13.0	.0	42	.0	-10.0	.0
43	.0	-7.5	.0	44	.0	-6.0	10.0	45	2.0	-28.0	10.0
46	2.0	-22.0	.0	47	2.0	-17.0	.0	48	2.0	-13.0	.0
49	2.0	-10.0	.0	50	2.0	-7.5	.0	51	1.0	-5.9	10.0
52	2.1	-5.6	10.0	53	3.0	-5.2	10.0	54	4.0	-28.0	10.0
55	4.0	-22.0	.0	56	4.0	-17.0	.0	57	4.0	-13.0	.0
58	4.0	-10.0	.0	59	4.0	-7.5	.0	60	3.9	-4.6	10.0
61	4.6	-3.9	10.0	62	6.0	-28.0	10.0	63	6.0	-22.0	.0
64	6.0	-17.0	.0	65	6.0	-13.0	.0	66	6.0	-10.0	.0
67	6.0	-7.5	.0	68	6.0	-5.0	.0	69	5.2	-3.0	10.0
70	5.6	-2.1	10.0	71	5.9	-1.0	10.0	72	6.0	.0	10.0
73	8.0	-28.0	.0	74	8.0	-22.0	.0	75	8.0	-17.0	.0
76	8.0	-13.0	.0	77	8.0	-10.0	.0	78	8.0	-7.5	.0
79	8.0	-5.0	.0	80	8.0	-2.5	.0	81	8.0	.0	.0

NODAL CONNECTIVITY

ELEM	N1	N2	N3	ELEM	N1	N2	N3	ELEM	N1	N2	N3	ELEM	N1	N2	N3
1	1	10	2	2	2	10	11	3	2	11	3	4	3	11	12
5	3	12	4	6	4	12	13	7	4	13	5	8	5	13	14
9	5	14	6	10	6	14	15	11	6	15	7	12	7	15	16
13	7	16	8	14	8	16	17	15	8	17	18	16	8	18	19
17	8	19	9	18	9	19	20	19	10	21	11	20	11	21	22
21	11	22	12	22	12	22	23	23	12	23	13	24	13	23	24
25	13	24	14	26	14	24	25	27	14	25	15	28	15	25	26
29	15	26	16	30	16	26	27	31	16	27	28	32	16	28	17
33	21	29	22	34	22	29	30	35	22	30	23	36	23	30	31
37	23	31	24	38	24	31	32	39	24	32	25	40	25	32	33
41	25	33	26	42	26	33	34	43	26	34	35	44	26	35	27
45	34	36	35	46	29	38	30	47	30	38	39	48	30	39	31
49	31	39	40	50	31	40	32	51	32	40	41	52	32	41	33
53	33	41	42	54	33	42	34	55	34	42	43	56	34	43	37
57	34	37	36	58	43	44	37	59	38	45	46	60	38	46	39
61	39	46	47	62	40	39	47	63	40	47	48	64	41	40	48
65	41	48	49	66	42	41	49	67	42	49	50	68	43	42	50
69	43	50	51	70	43	51	44	71	51	50	52	72	45	54	55
73	45	55	46	74	46	55	56	75	47	46	56	76	47	56	57
77	48	47	57	78	48	57	58	79	49	48	58	80	49	58	59
81	50	49	59	82	50	59	53	83	52	50	53	84	53	59	60
85	54	62	63	86	55	54	63	87	55	63	64	88	56	55	64
89	56	64	65	90	57	56	65	91	57	65	66	92	58	57	66
93	58	66	67	94	59	58	67	95	59	67	68	96	60	59	68
97	60	68	61	98	61	68	69	99	62	73	74	100	63	62	74
101	63	74	75	102	64	63	75	103	64	75	76	104	65	64	76
105	65	76	77	106	66	65	77	107	66	77	78	108	67	66	78
109	67	78	79	110	68	67	79	111	68	79	80	112	68	80	69
113	69	80	70	114	70	80	71	115	71	80	81	116	72	71	81

NUMBER OF NODES ON REFLECTION DOMAIN, NBR= 9

THEIR CONNECTIVITY ARE:

73 74 75 76 77 78 79 80 81

NUMBER OF NODES ON TRANSMISSION DOMAIN, NBT= 9

THEIR CONNECTIVITY ARE:

1 2 3 4 5 6 7 8 9

NUMBER OF SEGMENTS OF FREE SURFACE, NSGF= 2

NUMBER OF NODES ON 1-TH SEGMENT, NF(1)= 2

THEIR CONNECTIVITY ARE:

9 20

NUMBER OF NODES ON 2-TH SEGMENT, NF(2)= 2

THEIR CONNECTIVITY ARE:

```
72  81

- -- -- -- -- -- -- -- -- -- -- -- -- -- -- -- -- -- -- -- -- -- -- -- -- -- -

FOR REFLECTION DOMAIN:
          WATER DEPTH, HR=   28.00 HORIZONTAL EXTENT, XR=    8.00
FOR TRANSMISSION DOMAIN:
          WATER DEPTH, HT=   28.00 HORIZONTAL EXTENT, XT=   -8.00

- -- -- -- -- -- -- -- -- -- -- -- -- -- -- -- -- -- -- -- -- -- -- -- -- -- -

     NUMBER OF BODY FOR FORCE CALCULATION, NSGB=    1

        NUMBER OF NODES ON 1-TH BODY, NBOD( 1)=   19
                            LENGTH SCALE, SCL(I)=    6.00
                       REFERENCE CENTER, (XC,ZC)=     .00      .00
   THEIR CONNECTIVITY ARE:
20  19  18  17  28  27  35  36  37  44  51  52  53  60  61  69  70  71  72

- -- -- -- -- -- -- -- -- -- -- -- -- -- -- -- -- -- -- -- -- -- -- -- -- -- -

                         BANDWITH, NBD=  13

- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -

----------------------------------------
  WAVE PERIOD, WAVT=   12.00 SECONDS
----------------------------------------

- - - - - - - - - - - - - - - - - - - -
- - -  THE SOLUTION OF THE SYSTEM   - -
- - - - - - - - - - - - - - - - - - - -

                      NODAL-POTENTIAL
       NODE   REAL-PART  IMAGE-PART   ABS-VALUE  PHASE

         1       .6329       .0459       .6346   .072
         2       .6428       .0504       .6448   .078
         3       .6660       .0644       .6691   .096
         4       .6933       .0842       .6984   .121
         5       .7197       .1098       .7280   .151
         6       .7461       .1390       .7589   .184
```

7	.7775	.1767	.7974	.223
8	.8202	.2212	.8495	.263
9	.8762	.2532	.9121	.281
10	.6459	-.0053	.6459	-.008
11	.6557	-.0022	.6557	-.003
12	.6780	.0080	.6781	.012
13	.7040	.0243	.7044	.034
14	.7276	.0467	.7291	.064
15	.7499	.0766	.7538	.102
16	.7756	.1235	.7853	.158
17	.8011	.1544	.8158	.190
18	.8215	.1890	.8429	.226
19	.8462	.2143	.8729	.248
20	.8719	.2286	.9014	.256
21	.6569	-.0575	.6594	-.087
22	.6665	-.0559	.6689	-.084
23	.6883	-.0503	.6902	-.073
24	.7130	-.0396	.7141	-.055
25	.7341	-.0228	.7344	-.031
26	.7514	.0032	.7515	.004
27	.7712	.0593	.7735	.077
28	.7832	.1110	.7910	.141
29	.6657	-.1099	.6747	-.164
30	.6753	-.1102	.6843	-.162
31	.6970	-.1098	.7056	-.156
32	.7212	-.1068	.7290	-.147
33	.7406	-.0993	.7472	-.133
34	.7545	-.0844	.7592	-.111
35	.7637	.0055	.7638	.007
36	.7607	-.0532	.7626	-.070
37	.7616	-.1188	.7708	-.155
38	.6724	-.1624	.6918	-.237
39	.6822	-.1647	.7017	-.237
40	.7042	-.1700	.7244	-.237
41	.7288	-.1760	.7497	-.237
42	.7485	-.1808	.7701	-.237
43	.7624	-.1841	.7843	-.237
44	.7665	-.1851	.7886	-.237
45	.6770	-.2142	.7101	-.306
46	.6869	-.2186	.7209	-.308
47	.7098	-.2298	.7460	-.313
48	.7358	-.2450	.7756	-.321
49	.7580	-.2626	.8022	-.334
50	.7758	-.2852	.8266	-.352
51	.7757	-.2525	.8158	-.315
52	.7892	-.3211	.8520	-.386
53	.8052	-.3844	.8923	-.445
54	.6793	-.2652	.7292	-.372
55	.6896	-.2715	.7411	-.375
56	.7136	-.2882	.7696	-.384
57	.7419	-.3117	.8048	-.398
58	.7678	-.3399	.8396	-.417
59	.7918	-.3759	.8765	-.443
60	.8250	-.4447	.9372	-.494

61	.8489	-.5050	.9877	-.537
62	.6794	-.3147	.7487	-.434
63	.6900	-.3227	.7617	-.437
64	.7155	-.3444	.7941	-.449
65	.7463	-.3745	.8350	-.465
66	.7761	-.4098	.8777	-.486
67	.8061	-.4522	.9243	-.511
68	.8435	-.5144	.9880	-.548
69	.8772	-.5594	1.0404	-.568
70	.9064	-.6058	1.0902	-.589
71	.9380	-.6445	1.1381	-.602
72	.9684	-.6721	1.1787	-.607
73	.6769	-.3623	.7677	-.491
74	.6879	-.3717	.7819	-.495
75	.7150	-.3978	.8182	-.506
76	.7481	-.4323	.8640	-.524
77	.7815	-.4722	.9131	-.544
78	.8157	-.5160	.9652	-.564
79	.8571	-.5713	1.0300	-.588
80	.9119	-.6392	1.1136	-.611
81	.9781	-.7003	1.2030	-.621

COEFFICIENTS FOR REFLECTION DOMAIN

COEF	REAL-PART	IMAGE-PART	ABS-VALUE	PHASE
1	-.0436	-.3319	.3347	-1.701
2	-.0766	-.0995	.1255	-2.227
3	-.0144	-.1196	.1204	-1.691
4	.0088	-.1285	.1288	-1.50[illegible]
5	.0302	-.1431	.1463	-1.363
6	.0530	-.1554	.1642	-1.242

COEFFICIENTS FOR TRANSMISSION DOMAIN

COEF	REAL-PART	IMAGE-PART	ABS-VALUE	PHASE
1	.9334	-.1243	.9416	-.132
2	-.1021	.1429	.1757	2.191
3	-.0430	.1338	.1405	1.882
4	-.0191	.1321	.1335	1.714
5	-.0018	.1376	.1377	1.584
6	.0197	.1415	.1429	1.432

WAVE FORCES: INDX,1=FX,2=FZ,3=MY

INDX	REAL-PART	IMAGE-PART	ABS-VALUE	PHASE
1	-.0711	.6551	.6589	1.679
2	1.5950	-.3855	1.6409	-.237
3	-.0014	-.0009	.0017	-2.566

WAVE PERIOD, WAVT=   6.00 SECONDS

- - - THE SOLUTION OF THE SYSTEM - -

NODE	NODAL-POTENTIAL REAL-PART	IMAGE-PART	ABS-VALUE	PHASE
1	.1117	-.0860	.1410	-.656
2	.1171	-.0828	.1434	-.615
3	.1288	-.0682	.1457	-.487
4	.1420	-.0418	.1481	-.286
5	.1536	-.0023	.1536	-.015
6	.1653	.0474	.1720	.279
7	.1811	.1161	.2151	.570
8	.2105	.2070	.2952	.777
9	.2675	.2999	.4018	.842
10	.1244	-.1043	.1623	-.698
11	.1314	-.1042	.1677	-.670
12	.1469	-.0985	.1769	-.591
13	.1643	-.0824	.1838	-.465
14	.1780	-.0532	.1858	-.290
15	.1887	-.0098	.1889	-.052
16	.1983	.0610	.2074	.298
17	.2157	.1187	.2462	.503
18	.2268	.1754	.2868	.658
19	.2477	.2271	.3361	.742
20	.2757	.2681	.3846	.772
21	.1356	-.1228	.1829	-.736
22	.1442	-.1264	.1918	-.720
23	.1640	-.1314	.2102	-.675
24	.1862	-.1296	.2268	-.608
25	.2029	-.1151	.2333	-.516
26	.2128	-.0846	.2290	-.378
27	.2149	-.0110	.2152	-.051
28	.2108	.0553	.2179	.257
29	.1448	-.1404	.2017	-.770
30	.1552	-.1483	.2147	-.763
31	.1797	-.1655	.2443	-.744
32	.2080	-.1817	.2762	-.718
33	.2302	-.1878	.2971	-.684
34	.2432	-.1792	.3021	-.635

35	.2246	-.0758	.2370	-.325
36	.2404	-.1438	.2801	-.539
37	.2630	-.2204	.3432	-.697
38	.1517	-.1562	.2178	-.800
39	.1639	-.1687	.2353	-.800
40	.1935	-.1992	.2777	-.800
41	.2294	-.2362	.3293	-.800
42	.2608	-.2685	.3743	-.800
43	.2842	-.2926	.4079	-.800
44	.2914	-.3000	.4182	-.800
45	.1559	-.1693	.2301	-.826
46	.1698	-.1863	.2521	-.831
47	.2045	-.2299	.3077	-.844
48	.2492	-.2889	.3816	-.859
49	.2929	-.3507	.4570	-.875
50	.3339	-.4150	.5326	-.893
51	.3272	-.3873	.5070	-.869
52	.3725	-.4870	.6131	-.918
53	.4227	-.5903	.7261	-.949
54	.1570	-.1785	.2378	-.849
55	.1723	-.1995	.2636	-.858
56	.2117	-.2554	.3317	-.879
57	.2651	-.3350	.4272	-.901
58	.3224	-.4256	.5339	-.922
59	.3841	-.5298	.6544	-.943
60	.4832	-.7073	.8566	-.971
61	.5581	-.8461	1.0136	-.988
62	.1547	-.1832	.2398	-.869
63	.1709	-.2071	.2685	-.881
64	.2141	-.2731	.3470	-.906
65	.2745	-.3692	.4601	-.931
66	.3437	-.4837	.5934	-.953
67	.4238	-.6203	.7513	-.971
68	.5363	-.8167	.9770	-.990
69	.6506	-1.0093	1.2009	-.998
70	.7491	-1.1782	1.3962	-1.004
71	.8642	-1.3686	1.6186	-1.008
72	.9821	-1.5582	1.8419	-1.008
73	.1489	-.1825	.2355	-.887
74	.1652	-.2080	.2656	-.900
75	.2107	-.2812	.3513	-.928
76	.2748	-.3873	.4749	-.954
77	.3519	-.5178	.6261	-.974
78	.4426	-.6727	.8052	-.989
79	.5668	-.8852	1.0511	-1.001
80	.7511	-1.1955	1.4119	-1.010
81	1.0029	-1.6039	1.8916	-1.012

COEFFICIENTS FOR REFLECTION DOMAIN

COEF	REAL-PART	IMAGE-PART	ABS-VALUE	PHASE
1	-.3837	-.8337	.9177	-2.002

2	-.0699	.0454	.0834	2.565
3	-.0084	-.0541	.0548	-1.725
4	.0416	-.1225	.1294	-1.243
5	.0898	-.1914	.2114	-1.132
6	.1470	-.2702	.3076	-1.073

COEFFICIENTS FOR TRANSMISSION DOMAIN

COEF	REAL-PART	IMAGE-PART	ABS-VALUE	PHASE
1	.3561	-.1649	.3925	-.434
2	-.1047	.1348	.1707	2.231
3	-.0918	.1585	.1832	2.096
4	-.0660	.1508	.1646	1.984
5	-.0474	.1528	.1600	1.872
6	-.0278	.1590	.1615	1.744

WAVE FORCES: INDX,1=FX,2=FZ,3=MY

INDX	REAL-PART	IMAGE-PART	ABS-VALUE	PHASE
1	-.4223	1.0905	1.1694	1.940
2	.7025	-.7240	1.0088	-.800
3	-.0012	-.0033	.0035	-1.926

- -

*********CPU TIME, DT= 1.9402E+00 SECONDS **********
/

APPENDIX B: WAVERTF PROGRAM LISTING

The program WAVERTF (WAVE Reflection, Transmission, and Forces) is written in FORTRAN Extended IV or FORTRAN V. All FORTRAN callable subroutines are self-contained in the program. FORTRAN callable functions that are used are only those that the system recognizes. The program is listed on the following pages.

```
      PROGRAM WAVER F(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C  WES-CERC PROGRAM. HSCHEN. 1984
      PARAMETER(NELE=116,NNOD=81,NBD=13,NKR=6,NKT=6,NBR=9,NBT=9)
      PARAMETER(NFMX=2,NSGF=2,NBODMX=19,NSGB=1)
      COMPLEX SYSK(NNOD,NBD),SYSQ(NNOD),SYSDR(NKR),SYSKR(NKR,NBR),
     1SYSQR(NKR),SYSDT(NKT),SYSKT(NKT,NBR),SYSQT(NKT),EPSN(NNOD),
     2XLAMD(NNOD)
      DIMENSION TITLE(20),ICON(3,NELE),X(NNOD),Z(NNOD),INBR(NBR),
     1INBT(NBT),NF(NSGF),INF(NFMX,NSGF),WKR(NKR),WKHR(NKR),WKT(NKT),
     2WKHT(NKT),SCL(NSGB),XC(NSGB),ZC(NSGB),NBOD(NSGB),
     3INBOD(NBODMX,NSGB),BETA(NNOD)
      DATA NNODOT,NKROT,NKTOT,IFORCE/81,6,6,1/
      DATA GAMA,VISCO,G,TOLR/-0.7853916,1.E-6,9.80621,1.E-4/
CC      CALL CPTIME(CA)
      PI=3.1415926
    2 FORMAT(20I4)
    4 FORMAT(8F10.4)
      READ(5,8) TITLE
    8 FORMAT(20A4)
      WRITE(6,9) TITLE
    9 FORMAT(//26('- -')/20A4/26('- -'))
      WRITE(6,10) NELE,NNOD,NKR,NKT,NFMX,IFORCE,NBODMX
   10 FORMAT(//24X,'TOTAL NUMBER OF ELEMENTS, NELE=',I4/
     1 27X,'TOTAL NUMBER OF NODES, NNOD=',I4/6X,'TOTAL NUMBER OF EIGEN V
     2ALUES FOR REFLECTION, NKR=',I4/4X,'TOTAL NUMBER OF EIGEN VALUES FO
     3R TRANSMISSION, NKT=',I4/14X,'MAX NODAL NO. ALLOWED IN A SEGMENT,
     4NFMX=',I4/10X,'INDEX FOR FORCE CALCULATION (NONE=0), IFORCE=',I4/
     5 15X,'MAX NODAL NO. ALLOWED IN A BODY, NBODMX=',I4/)
      READ(5,12) (I,X(I),Z(I),BETA(I),J=1,NNOD)
   12 FORMAT(3(I4,3F6.0))
      CALL DASHLN(26)
      WRITE(6,15)
      WRITE(6,16) (J,X(J),Z(J),BETA(J),J=1,NNOD)
   15 FORMAT(/16X,'(X,Z) COORDINATE AND FRICTION COEF FOR EACH NOD',
     1  /1X,3('NOD      X         Z      FRIC ')/)
   16 FORMAT(3(I4,2F8.1,F6.1))
      READ(5,2) (J,(ICON(I,J),I=1,3),L=1,NELE)
      CALL DASHLN(26)
      WRITE(6,20)
   20 FORMAT(/28X,'NODAL CONNECTIVITY'/1X,4('ELEM   N1   N2   N3 ')/)
      WRITE(6,21) (L,(ICON(I,L),I=1,3),L=1,NELE)
   21 FORMAT(4(4I5))
      CALL DASHLN(26)
      NNBR=NNOD-NBR
      DO 23 I=1,NBR
      INBR(I)=NNBR+I
   23 CONTINUE
      WRITE(6,25) NBR
   25 FORMAT(/12X,' NUMBER OF NODES ON REFLECTION DOMAIN, NBR=',I4/
     1    27X,'THEIR CONNECTIVITY ARE:')
      WRITE(6,2) (INBR(I),I=1,NBR)
      DO 26 I=1,NBT
      INBT(I)=I
```

```
   26 CONTINUE
      WRITE(6,28) NBT
   28 FORMAT(//10X,' NUMBER OF NODES ON TRANSMISSION DOMAIN, NBT=',I4/
     1   27X,'THEIR CONNECTIVITY ARE:')
      WRITE(6,2) (INBT(I),I=1,NBT)
      WRITE(6,30) NSGF
   30 FORMAT(//13X,' NUMBER OF SEGMENTS OF FREE SURFACE, NSGF=',I4)
      DO 35 I=1,NSGF
      READ(5,2) JJ,(INF(J,I),J=1,JJ)
      NF(I)=JJ
      WRITE(6,32) I,I,NF(I)
   32 FORMAT(/14X,' NUMBER OF NODES ON',I2,'-TH SEGMENT, NF(',I2,
     1   ')=',I4/27X,'THEIR CONNECTIVITY ARE:')
      WRITE(6,2) (INF(J,I),J=1,JJ)
   35 CONTINUE
      CALL DASHLN(26)
      HR=-Z(NNBR+1)
      XR=X(NNBR+1)
      HT=-Z(1)
      XT=X(1)
      WRITE(6,40) HR,XR,HT,XT
   40 FORMAT(/2X,'FOR REFLECTION DOMAIN:'/13X,' WATER DEPTH, HR=',F8.2,
     1' HORIZONTAL EXTENT, XR=',F8.2,/2X,'FOR TRANSMISSION DOMAIN:'/
     213X,' WATER DEPTH, HT=',F8.2,' HORIZONTAL EXTENT, XT=',F8.2/)
      IF(IFORCE.EQ.0) GOTO 50
      CALL DASHLN(26)
      WRITE(6,42) NSGB
   42 FORMAT(//7X,' NUMBER OF BODY FOR FORCE CALCULATION, NSGB= ',I4)
      DO 45 I=1,NSGB
      READ(5,2) JJ,(INBOD(J,I),J=1,JJ)
      READ(5,4) SCL(I),XC(I),ZC(I)
      NBOD(I)=JJ
      WRITE(6,44) I,I,NBOD(I),SCL(I),XC(I),ZC(I)
   44 FORMAT(/12X,'NUMBER OF NODES ON',I2,'-TH BODY, NBOD(',I2,')= ',I4,
     1 /30X,  'LENGTH SCALE, SCL(I)=',F8.2/25X,'REFERENCE CENTER, (XC,ZC
     2)=',2F8.2/6X,'THEIR CONNECTIVITY ARE:')
      WRITE(6,2) (INBOD(J,I),J=1,JJ)
   45 CONTINUE
      CALL DASHLN(26)
   50 CONTINUE
      CALL BAND(ICON,NELE,NBR,NBT,NBD)
  400 CALL DATAIN(IGO,OMGSG,OMGA,G)
      IF(IGO.EQ.0) GOTO 404
C     CALL DASHLN(26)
      CALL EIGVAL(WKR,WKHR,NKR,HR,OMGSG,PI,TOLR)
      CALL EIGVAL(WKT,WKHT,NKT,HT,OMGSG,PI,TOLR)
C     CALL DASHLN(26)
      CALL FRICTI(EPSN,XLAMD,BETA,NNOD,OMGA,GAMA,VISCO,HR,WKR,PI)
      CALL LAPLAC(SYSK,ICON,NELE,NNOD,NBD,X,Z,XLAMD)
      CALL SURFAC(SYSK,NNOD,NBD,NSGF,NF,INF,NFMX,X,OMGSG)
      CALL REFTRA(SYSDR,WKR,WKHR,NKR,OMGSG,XR,1.0)
      CALL REFTRA(SYSDT,WKT,WKHT,NKT,OMGSG,XT,-1.0)
      CALL HYBRID(SYSKR,NNOD,WKR,WKHR,NKR,Z,XR,HR,INBR,NBR,1.0)
      CALL HYBRID(SYSKT,NNOD,WKT,WKHT,NKT,Z,XT,HT,INBT,NBT,-1.0)
```

```
      CALL CZERO(SYSQR,NKR)
      SYSQR(1)=(0.0,-1.0)*WKR(1)*AM(OMGSG,WKR(1),WKHR(1),1)
      CALL CZERO(SYSQT,NKT)
      CALL BOUNDA(SYSQ,NNOD,WKR(1),WKHR(1),NBR,Z,XR,HR)
      CALL REDUCE(SYSK,SYSQ,SYSKR,SYSDR,SYSQR,NNOD,NBD,NKR,NBR,2)
      CALL REDUCE(SYSK,SYSQ,SYSKT,SYSDT,SYSQT,NNOD,NBD,NKT,NBT,1)
      CALL CSIMQ(SYSK,SYSQ,NNOD,NBD)
      CALL SOLVE(SYSQ,SYSKR,SYSDR,SYSQR,NNOD,NKR,NBR,2)
      CALL SOLVE(SYSQ,SYSKT,SYSDT,SYSQT,NNOD,NKT,NBT,1)
      IF(NNODOT.LT.1) GOTO 67
      WRITE(6,60)
   60 FORMAT(////20(' -')/' - - -  THE SOLUTION OF THE SYSTEM   - -'/
     1  20(' -')//27X,'NODAL-POTENTIAL'/12X,'NODE   REAL-PART  IMAGE-PAR
     2T   ABS-VALUE  PHASE'/)
      DO 65 I=1,NNOD
      CALL OUTCPX(I,SYSQ(I))
   65 CONTINUE
   67 IF(NKROT.LT.1) GOTO 77
      WRITE(6,70)
   70 FORMAT(///18X,'COEFFICIENTS FOR REFLECTION DOMAIN')
      WRITE(6,72)
   72 FORMAT(12X,'COEF   REAL-PART  IMAGE-PART   ABS-VALUE  PHASE'/)
      DO 75 I=1,NKROT
      CALL OUTCPX(I,SYSQR(I))
   75 CONTINUE
   77 IF(NKTOT.LT.1) GOTO 87
      WRITE(6,80)
   80 FORMAT(///17X,'COEFFICIENTS FOR TRANSMISSION DOMAIN')
      WRITE(6,72)
      DO 85 I=1,NKTOT
      CALL OUTCPX(I,SYSQT(I))
   85 CONTINUE
   87 IF(IFORCE.NE.0) CALL FORCE(SYSQ,X,Z,NNOD,NBGDMX,NSGB,NBOD,INBOD,
     1 SCL,XC,ZC,EPSN,XLAMD,OMGA)
      CALL DASHLN(26)
      GOTO 400
  404 CONTINUE
CC    CALL CPTIME(CB)
CC    DT=CB-CA
CC    WRITE(6,500) DT
CC500 FORMAT(/9('*'),'CPU TIME, DT=',1PE12.4,' SECONDS ',10('*'))
      STOP
      END
      SUBROUTINE DASHLN(N)
C - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
C - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
      DATA DASH/3H- -/
      WRITE(6,4) (DASH,I=1,N)
    4 FORMAT(//1X,42A3)
      RETURN
      END
      SUBROUTINE ZERO(A,N)
C - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
C - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
```

```
      DIMENSION A(N)
      DO 4 I=1,N
      A(I)=0.0
    4 CONTINUE
      RETURN
      END
      SUBROUTINE CZERO(A,N)
C ----------------------------------------
C ----------------------------------------
      COMPLEX A(N)
      DO 4 I=1,N
      A(I)=(0.0,0.0)
    4 CONTINUE
      RETURN
      END
      SUBROUTINE BAND(ICON,NELE,NBR,NBT,NBD)
C ----------------------------------------
C ----------------------------------------
      DIMENSION ICON(3,NELE)
      KMX=1
      DO 8 L=1,NELE
      IMX=ICON(1,L)
      IMN=ICON(1,L)
      DO 4 K=2,3
      IF(ICON(K,L).LT.IMN) IMN=ICON(K,L)
      IF(ICON(K,L).GT.IMX) IMX=ICON(K,L)
    4 CONTINUE
      KK=IMX-IMN
      IF(KK.GT.KMX) KMX=KK
    8 CONTINUE
      NBD=KMX+1
      NBD=AMAX0(NBD,NBR,NBT)
      WRITE(6,10) NBD
   10 FORMAT(//27X,'BANDWITH, NBD=',I4//40('- '))
      RETURN
      END
      SUBROUTINE DATAIN(IGO,OMGSG,OMGA,G)
C ----------------------------------------
C OMGSG=(WAVE FREQUENCY)**2/GRAVITY CONSTANT.
C ----------------------------------------
      READ(5,4) IGO,WAVT
    4 FORMAT(I10,F10.1)
      IF(IGO.EQ.0) RETURN
      WRITE(6,8) WAVT
    8 FORMAT(//20('--')/3X,'WAVE PERIOD, WAVT=',F8.2,' SECONDS',/20('--'
     1)//)
      OMGA=6.2831853/WAVT
      OMGSG=OMGA*OMGA/G
      RETURN
      END
      SUBROUTINE EIGVAL(WK,WKH,NK,H,OMGSG,PI,TOLR)
C ----------------------------------------
C ----------------------------------------
      DIMENSION WK(NK),WKH(NK)
```

```
C      WRITE(6,4)
C    4 FORMAT(//45('-')/4('-'),' EIGEN VALUES OF DISPERSION EQUATION ',
C     14('-')/45('-')//19X,'FIRST NUMBER IS REAL, OTHERS ARE IMAGINARY'/)
       C=OMGSG*H
       XJ=C
    10 XI=XJ
       XJ=C/TANH(XI)
       IF(ABS(XI-XJ).GT.TOLR*XJ) GOTO 10
       WKH(1)=XJ
       WK(1)=WKH(1)/H
       IF(NK.LE.1) GOTO 25
       DO 20 I=2,NK
       XJ=(I-1)*PI
       DX=XJ
    15 XI=XJ
       XJ=ATAN(-C/XI)+DX
       IF(ABS(XI-XJ).GT.TOLR*XJ) GOTO 15
       WKH(I)=XJ
       WK(I)=WKH(I)/H
    20 CONTINUE
    25 CONTINUE
C      WRITE(6,35)
C   35 FORMAT(11X,'WAVE NUMBER, WK, ARE:')
C      WRITE(6,37) (WK(I),I=1,NK)
C   37 FORMAT(8F10.4)
C      WRITE(6,40)
C   40 FORMAT('  WAVE NUMBER * DEPTH, WKH, ARE:')
C      WRITE(6,37) (WKH(I),I=1,NK)
       RETURN
       END
       SUBROUTINE FRICTI(EPSN,XLAMD,BETA,NNOD,OMGA,GAMA,VISCO,H,WK,PI)
C    - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
C    - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
       COMPLEX EPSN(NNOD),XLAMD(NNOD),TM,WI,EXPG
       DIMENSION BETA(NNOD)
       WI=(0.0,1.0)/OMGA
       EXPG=COS(GAMA)+(0.0,1.0)*SIN(GAMA)
       TM=EXPG*SQRT(VISCO*OMGA)
       DO 20 I=1,NNOD
       EPSN(I)=TM*BETA(I)
       XLAMD(I)=1.0/(1.0+WI*EPSN(I))
    20 CONTINUE
       RETURN
       END
       SUBROUTINE EIGVAL1(WKH,H,OMGSG,TOLR)
C    - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
C    - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
       C=OMGSG*H
       XJ=C
    10 XI=XJ
       XJ=C/TANH(XI)
       IF(ABS(XI-XJ).GT.TOLR*XJ) GOTO 10
       WKH=XJ
       RETURN
```

```
      END
      SUBROUTINE ELMK(MEL,X,Y,ELK)
C - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
C   GENERATION OF TIANGULAR ELEMENT MATRIX, ELK.
C - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
      DIMENSION X(3),Y(3),ELK(3,3)
      CALL ZERO(ELK,9)
      B1=Y(2)-Y(3)
      B2=Y(3)-Y(1)
      B3=Y(1)-Y(2)
      C1=X(3)-X(2)
      C2=X(1)-X(3)
      C3=X(2)-X(1)
      AREA=0.5*(B1*C2-B2*C1)
      IF(AREA.GT.0.0) GOTO 20
      WRITE(6,10) MEL,AREA
   10 FORMAT(' ****',I5,'-TH ELEMENT, AREA=',F10.5)
      STOP
   20 A4=4.*AREA
      ELK(1,1)=(B1*B1+C1*C1)/A4
      ELK(1,2)=(B1*B2+C1*C2)/A4
      ELK(1,3)=(B1*B3+C1*C3)/A4
C     ELK(2,1)=ELK(1,2)
      ELK(2,2)=(B2*B2+C2*C2)/A4
      ELK(2,3)=(B2*B3+C2*C3)/A4
C     ELK(3,1)=ELK(1,3)
C     ELK(3,2)=ELK(2,3)
      ELK(3,3)=(B3*B3+C3*C3)/A4
      RETURN
      END
      SUBROUTINE LAPLAC(SYSK,ICON,NELE,NNOD,NBD,X,Y,XLAMD)
C - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
C   ASSEMBLE ELK INTO SYSK FOR LAPLACIAN.
C - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
      COMPLEX SYSK(NNOD,NBD),XLAMD(NNOD),TM
      DIMENSION ICON(3,NELE),X(NNOD),Y(NNOD),IE(3),XE(3),YE(3),ELK(3,3)
      NN=NNOD*NBD
      CALL CZERO(SYSK,NN)
      DO 40 L=1,NELE
      TM=(0.0,0.0)
      DO 10 J=1,3
      IE(J)=ICON(J,L)
      XE(J)=X(IE(J))
      YE(J)=Y(IE(J))
      TM=TM+XLAMD(IE(J))
   10 CONTINUE
      TM=TM/3.
      CALL ELMK(L,XE,YE,ELK)
      DO 30 I=1,3
      DO 20 J=I,3
      JI=IE(J)-IE(I)
      IF(JI.GE.0) GOTO 15
      JI1=-JI+1
      SYSK(IE(J),JI1)=SYSK(IE(J),JI1)+TM*ELK(I,J)
```

```
      GOTO 20
   15 JI1=JI+1
      SYSK(IE(I),JI1)=SYSK(IE(I),JI1)+TM*ELK(I,J)
   20 CONTINUE
   30 CONTINUE
   40 CONTINUE
      RETURN
      END
      SUBROUTINE SURFAC(SYSK,NNOD,NBD,NSGF,NF,INF,NFMX,X,OMGSG)
C - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
C - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
      COMPLEX SYSK(NNOD,NBD)
      DIMENSION X(NNOD),NF(NSGF),INF(NFMX,NSGF)
      EX=OMGSG/3.
      DO 30 I=1,NSGF
      JJ=NF(I)
      DO 20 J=2,JJ
      J1=INF(J-1,I)
      J2=INF(J,I)
      EXL=EX*(X(J2)-X(J1))
      SYSK(J1,1)=SYSK(J1,1)-EXL
      SYSK(J2,1)=SYSK(J2,1)-EXL
      J12=J1-J2
      IF(J12.GE.0) GOTO 10
      JA=-J12+1
      SYSK(J1,JA)=SYSK(J1,JA)-EXL/2.
      GOTO 20
   10 JA=J12+1
      SYSK(J2,JA)=SYSK(J2,JA)-EXL/2.
   20 CONTINUE
   30 CONTINUE
      RETURN
      END
      SUBROUTINE REFTRA(SYSD,WK,WKH,NK,OMGSG,XX,SGN)
C - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
C   ASSEMBLE ELEMENT OF COEFFICIENT TYPE.
C   USE SGN=1.0 FOR REFLECTION, SGN=-1.0 FOR TRANSIMISSION.
C - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
      COMPLEX SYSD(NK),CX
      DIMENSION WK(NK),WKH(NK)
      CALL CZERO(SYSD,NK)
      DO 20 I=1,NK
      XK=SGN*2.*WK(I)*XX
      IF(I.GT.1) GOTO 10
      CX=(0.0,1.0)*COS(XK)-SIN(XK)
      SYSD(I)=CX*WK(I)*AM(OMGSG,WK(I),WKH(I),I)
      GOTO 20
   10 SYSD(I)=-EXP(-XK)*WK(I)*AM(OMGSG,WK(I),WKH(I),I)
   20 CONTINUE
      RETURN
      END
      FUNCTION AM(OMGSG,WK,WKH,INDX)
C - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
C   INTEGRATION OF (COSHK(Z+H)/COSHKH)**2 FROM -H TO 0.
```

```
C     OMGSG=OM*OM/G, WHERE OM=WAVE CIRCULAR FREQUENCY AND G=GRAVITATION
C                  CONSTANT.
C     WK=WAVE NUMBER
C     WKH=WK*H, WHERE H=WATER DEPTH
C     INDX=1, PROPAGATING MODE, OTHERWISE EVANESCENT MODES.
C - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
      IF(INDX.EQ.1) GOTO 4
      AM=0.5*(WKH/COS(WKH)**2-OMGSG/WK)/WK
      RETURN
    4 AM=0.5*(WKH/COSH(WKH)**2+OMGSG/WK)/WK
      RETURN
      END
      SUBROUTINE HYBRID(SYSKRT,NNOD,WK,WKH,NK,Z,XX,H,INB,NB,SGN)
C - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
C    USE SGN=1.0 FOR REFLECTION, SGN=-1.0 FOR TRANSIMISSION.
C - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
      COMPLEX SYSKRT(NK,NB),CX,TM
      DIMENSION Z(NNOD),WK(NK),WKH(NK),INB(NB)
      NN=NK*NB
      CALL CZERO(SYSKRT,NN)
      DO 40 I=1,NK
      EX=SGN*WK(I)*XX
      WKZ2=0.0
      CH2=1.0
      IF(I.EQ.1) GOTO 10
      CX=EXP(-EX)/COS(WKH(I))
      GOTO 14
   10 CX=((0.0,1.0)*COS(EX)-SIN(EX))/COSH(WKH(I))
   14 DO 30 J=2,NB
      WKZ1=WKZ2
      WKZ2=WK(I)*(Z(INB(J))+H)
      CH1=CH2
      IF(I.EQ.1) GOTO 24
      CH2=COS(WKZ2)
      GOTO 28
   24 CH2=COSH(WKZ2)
   28 TM=CX*(CH2-CH1)/(WKZ2-WKZ1)
      SYSKRT(I,J-1)=SYSKRT(I,J-1)-TM
      SYSKRT(I,J)=SYSKRT(I,J)+TM
   30 CONTINUE
      IF(I.EQ.1) SYSKRT(I,NB)=SYSKRT(I,NB)-CX*SINH(WKH(I))
      IF(I.NE.1) SYSKRT(I,NB)=SYSKRT(I,NB)+CX*SIN(WKH(I))
   40 CONTINUE
      RETURN
      END
      SUBROUTINE BOUNDA(SYSQ,NNOD,WK1,WKH1,NB,Z,XX,H)
C - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
C - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
      COMPLEX SYSQ(NNOD),CX,TM
      DIMENSION Z(NNOD)
      CALL CZERO(SYSQ,NNOD)
      NN=NNOD-NB
      XK=-WK1*XX
      CX=((0.0,1.0)*COS(XK)-SIN(XK))/COSH(WKH1)
```

```
      WKZ2=0.0
      CH2=1.0
      DO 10 J=2,NB
      NJ=NN+J
      WKZ1=WKZ2
      WKZ2=WK1*(Z(NJ)+H)
      CH1=CH2
      CH2=COSH(WKZ2)
      TM=CX*(CH2-CH1)/(WKZ2-WKZ1)
      SYSQ(NJ-1)=SYSQ(NJ-1)-TM
      SYSQ(NJ)=SYSQ(NJ)+TM
   10 CONTINUE
      SYSQ(NNOD)=SYSQ(NNOD)-CX*SINH(WKH1)
      RETURN
      END
      SUBROUTINE REDUCE(SYSK,SYSQ,SYSKRT,SYSDRT,SYSQRT,NNOD,NBD,NK,NB,
     1                  INDX)
C  - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
C   INDX=1, MERGE FROM THE TOP OF THE MATRIX, OTHERWISE MERGE FROM THE
C         BOTTOM.
C  - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
      COMPLEX SYSK(NNOD,NBD),SYSQ(NNOD),SYSKRT(NK,NB),SYSDRT(NK),
     1        SYSQRT(NK),TM,TMO,TM1
      NN=NNOD-NB
      DO 30 I=1,NK
      TMO=SYSQRT(I)/SYSDRT(I)
      DO 20 J=1,NB
      JJ=J
      IF(INDX.NE.1) JJ=NN+J
      SYSQ(JJ)=SYSQ(JJ)-TMO*SYSKRT(I,J)
      TM1=SYSKRT(I,J)/SYSDRT(I)
      DO 10 K=1,J
      KK=K
      IF(INDX.NE.1) KK=NN+K
      JK=J-K+1
      TM=TM1*SYSKRT(I,K)
      SYSK(KK,JK)=SYSK(KK,JK)-TM
   10 CONTINUE
   20 CONTINUE
   30 CONTINUE
      RETURN
      END
      SUBROUTINE CSIMQ(A,B,NEQT,NBD)
C  - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
C  - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
      COMPLEX A,B
      DIMENSION A(NEQT,NBD),B(NEQT)
      TOLR=1.0E-30
      NNB=NEQT-NBD+1
      NB=NBD-1
      DO 24 I=1,NEQT
      IF(CABS(A(I,1)).LE.TOLR) GOTO 40
      A(I,1)=1.0/A(I,1)
      B(I)=B(I)*A(I,1)
```

```
      IF(NNB.GE.I) GOTO 10
      NB=NEQT-I
      IF(NB.LE.0) GOTO 24
   10 N1=1
      DO 14 J=1,NB
      N1=N1+1
   14 A(I,N1)=A(I,N1)*A(I,1)
      DO 20 J=1,NB
      IJ=I+J
      JJ=NB+1-J
      J1=J+1
      B(IJ)=B(IJ)-B(I)*A(I,J1)/A(I,1)
      DO 16 K=1,JJ
      KJ=K+J
   16 A(IJ,K)=A(IJ,K)-A(I,J1)*A(I,KJ)/A(I,1)
   20 CONTINUE
   24 CONTINUE
      J=NEQT
      N1=NEQT-1
      DO 34 K=1,N1
      NB=NBD-1
      J=J-1
      IF(NNB.GE.J) GOTO 28
      NB=NEQT-J
   28 DO 34 I=1,NB
      IJ=J+I
      JJ=I+1
      B(J)=B(J)-B(IJ)*A(J,JJ)
   34 CONTINUE
      RETURN
   40 WRITE(6,41) A(I,1),I
   41 FORMAT(' * ELEMENT OF DIAGONAL=',2E15.6,' AT',I4,'-TH ROW * ')
      STOP
      END
      SUBROUTINE SOLVE(SYSQ,SYSKRT,SYSDRT,SYSQRT,NNOD,NK,NB,INDX)
C - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
C  INDX=1, MERGE FROM THE TOP, OTHERWISE FROM THE BOTTOM.
C - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
      COMPLEX SYSQ(NNOD),SYSKRT(NK,NB),SYSDRT(NK),SYSQRT(NK),TM
      NN=NNOD-NB
      DO 20 I=1,NK
      DO 10 J=1,NB
      JJ=J
      IF(INDX.NE.1) JJ=NN+J
      TM=SYSQ(JJ)*SYSKRT(I,J)
      SYSQRT(I)=SYSQRT(I)-TM
   10 CONTINUE
      SYSQRT(I)=SYSQRT(I)/SYSDRT(I)
   20 CONTINUE
      RETURN
      END
      SUBROUTINE OUTCPX(I,CA)
C - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
C - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
```

```
      COMPLEX CA
      AR=REAL(CA)
      AI=AIMAG(CA)
      AT=ATAN2(AI,AR)
      AS=SQRT(AR*AR+AI*AI)
      WRITE(6,4) I,AR,AI,AS,AT
    4 FORMAT(8X,I7,1X,3F12.4,F7.3)
      RETURN
      ENll
      SUBROUTINE FORCE(SYSQ,X,Z,NNOD,NBODMX,NSGB,NBOD,INBOD,SCL,XC,ZC,
     1                   EPSN,XLAMD,OMGA)
C   - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
C   - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
      COMPLEX SYSQ(NNOD),TM,P1,P2,FX,FZ,VMY,EPSN(NNOD),XLAMl:(NNOD),T1,
     1        T2,TX,TZ,WI
      DIMENSION X(NNOD),Z(NNOD),NBOD(NSGB),INBOD(NBODMX,NSGB),SCL(NSGB),
     1  XC(NSGB),ZC(NSGB)
      WI=(0.0,1.0)/OMGA
      DO 30 I=1,NSGB
C     WRITE(6,5) I
C   5 FORMAT(///19X,'HYDRODYNAMIC FORCES FOR',I2,'-TH BODY')
C     WRITE(6,10)
C  10 FORMAT(//31X,'PRESSURE'/12X,'NODE    REAL-PART  IMAGE-PART   ABS-VA
C    1LUE  PHASE'/)
      JJ=NBOD(I)
      FX=(0.0,0.0)
      FZ=(0.0,0.0)
      VMY=(0.0,0.0)
      J1=INBOD(JJ,I)
      J2=INBOD(1,I)
      X1=X(J1)-XC(I)
      Z1=Z(J1)-ZC(I)
      X2=X(J2)-XC(I)
      Z2=Z(J2)-ZC(I)
C     AREAB=-0.5*(X2-X1)*(Z1+Z2)
      T2=-WI*EPSN(J2)*XLAMD(J2)
      P2=SYSQ(J2)
C     CALL OUTCPX(J2,P2)
      DO 20 J=2,JJ
      J1=J2
      X1=X2
      Z1=Z2
      P1=P2
      T1=T2
      J2=INBOD(J,I)
      X2=X(J2)-XC(I)
      Z2=Z(J2)-ZC(I)
      P2=SYSQ(J2)
      T2=-WI*EPSN(J2)*XLAMD(J2)
C     CALL OUTCPX(J2,P2)
      DX=X2-X1
      DZ=Z2-Z1
C     AREAB=AREAB-0.5*DX*(Z1+Z2)
      TM=0.5*(P1+P2)
```

```
      FX=FX-TM*DZ
      FZ=FZ+TM*DX
      TM=0.5*(T1+T2)*(P2-P1)
      TX=TM*DX
      TZ=TM*DZ
      FX=FX+TX
      FZ=FZ+TZ
      X11=X1*X1
      X12=X1*X2
      X22=X2*X2
      Z11=Z1*Z1
      Z12=Z1*Z2
      Z22=Z2*Z2
      TM=P1*(Z22+Z12-2.0*Z11)+P2*(2.0*Z22-Z12-Z11)
      TM=(TM+P1*(X22+X12-2.0*X11)+P2*(2.0*X22-X12-X11))/6.0
      VMY=VMY+TM
      VMY=VMY+(-TX*(Z2+Z1)+TZ*(X2+X1))/2.0
   20 CONTINUE
C     FX=FX/AREAB
C     FZ=FZ/AREAB
C     VMY=VMY/AREAB/SCL(I)
      FX=FX/SCL(I)
      FZ=FZ/SCL(I)
      VMY=VMY/SCL(I)/SCL(I)
      WRITE(6,24)
   24 FORMAT(//19X,'WAVE FORCES: INDX,1=FX,2=FZ,3=MY'/12X,'INDX   REAL-P
     1ART  IMAGE-PART   ABS-VALUE  PHASE'/)
      CALL OUTCPX(1,FX)
      CALL OUTCPX(2,FZ)
      CALL OUTCPX(3,VMY)
   30 CONTINUE
      RETURN
      END
/
```

APPENDIX C: NOTATION

a_o	Incident wave amplitude
A	Water region
B	Solid body
f_i	Wave force coefficient
F	Total force
g	Gravitational acceleration
h	Water depth
i	$\sqrt{-1}$
j	subscript $j=1,2$ represents the x and z components
$k_m^{\pm}$	Wave number of the evanescent modes $m \geq 1$
$k_o^{\pm}$	Wave number of the propagating mode
ℓ	Length
L	Length scale for the force normalization
m	Moment coefficients
M	Moment
n	Unit normal vector outward from the water region
p	Pressure
p_d	Hydrodynamic pressure
$\vec{r}$	Distance vector
R	Coefficients in Equation 14
t	Temporal coordinate
u	Flow velocity
U	Spatial part of the flow velocity
x	Horizontal direction
z	Vertical direction
α	Absorption coefficient
β	Friction coefficient
γ	Phase difference
ε	Friction coefficient of the solid or structural boundaries
ζ	Spatial part of the free surface displacement
η	Free surface displacement

λ	Associated with bottom friction and wave frequency given in Equation 6
ν	Water viscosity
Π	Functional
ρ	Water density
τ	Friction
ϕ	Velocity potential function
Φ	Spatial part of the velocity potential function
ω	Radian wave frequency
∂	Boundary curve: such as ∂A , ∂B , ∂F,...etc.
∇	Two-dimensional gradient operator
o	Subscript o indicates incident wave
+	Superscript + indicates the reflection water region
-	Superscript - indicates the transmission water region

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